9.1 & 9.2 Practice

1. Zach reads a newspaper report claiming that 12% of all adults in the United States are left-handed. He wonders if the proportion of lefties at his Ivy League school is really 12%. Zach chooses an SRS of 100 students and records whether each student is right- or left-handed.
   - Define the parameter and state the appropriate null hypothesis H_0 and alternative hypothesis H_a.
   - Check that the conditions for carrying out a one-sample z test for the population proportion p are met.

$H_0: p = .12$
$H_A: p \neq .12$

where p = the true proportion of lefties at Zach's ivy league school.

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2. In planning a study of the birth weights of babies whose mothers did not see a doctor before delivery, a researcher states the hypotheses as

$$H_0: \bar{x} = 1000 \text{ grams}$$
$$H_a: \bar{x} < 1000 \text{ grams}$$

Explain what's wrong with the stated hypotheses. *(Hint: there are two errors)* Then give correct hypotheses.

$H_0: \mu = 1000$ grams
$H_A: \mu < 1000$ grams
where μ = true mean birth weights of babies whose mothers did not see a dr. before delivery

3. Hemoglobin is a protein in red blood cells that carries oxygen from the lungs to body tissues. People with less than 12 grams of hemoglobin per deciliter of blood (g/dl) are anemic. A public health official in Jordan suspects that Jordanian children are at risk of anemia. He measures a random sample of 50 children.
   - Define the parameter and state the appropriate null hypothesis H_0 and alternative hypothesis H_a.
   - Check that the conditions for carrying out a significance test of the official's suspicion.

$H_0: \mu = 12$ g/dl
$H_A: \mu < 12$ g/dl.

where μ = the mean hemoglobin (g/dl) of blood for Jordanian children.

Random: stated Random sample of 50 children.
Normal: $n = 50 \geq 30$. CLT applies → sampling dist. is relatively normal
Indep.: $10(50) = 500$ More than 500 children in Jordan.

4. The sample mean hemoglobin level was 11.3 g/dl and the sample standard deviation was 1.6 g/dl. A significance test yields a P-value of 0.016. Here are the hypotheses for μ = true mean hemoglobin level:

$$H_0: \mu = 12 \text{ g/dl}$$
$$H_a: \mu < 12 \text{ g/dl}$$

(a) What conclusion would you make if $\alpha = 0.05$?

P value = .016 < α = .05 which means you can reject H_0
There is convincing evidence that the true mean hemoglobin level is less than 12 g/dl.

(b) If $\alpha = 0.01$?

P value = .016 > α = .01 → Fail to reject H_0
There is not convincing evidence that the true mean hemoglobin level is less than 12 g/dl

5. A student performs a test of $H_0: p = 0.5$ versus $H_a: p \neq 0.5$ and gets a P-value of 0.63. The student writes: "Because the P-value is greater than $\alpha = 0.05$, we accept H_0. The data provides convincing evidence that the null hypothesis is true." Explain what is wrong with this conclusion.

Fail to reject H_0. There is not convincing evidence that the true proportion is different than .50.

6. A state's Division of Motor Vehicles (DMV) claims that 60% of all teens pass their driving test on the first attempt. An investigative reporter examines an SRS of the DMV records for 125 teens; 86 of them passed the test on their first try.

(a) Is there convincing evidence at the 5% significance level that the DMV's claim is incorrect?? (State, Plan, Do, Conclude)

State: $\alpha = .05$

$H_0: p = .60$

$H_A: p \neq .60$

where p = true proportion of teens in the state who pass on the 1st attempt.

Plan: Use a 1-Prop Z test

Random: SRS of 125 stated

Indep: 10(125) = 1250 < pop. of teens

Normal

$n(p) \geq 10$ ✓ $125(.6) = 75$ $n(1-p) \geq 10$ $125(.4) = 50$

DO:

$\hat{p} = \frac{86}{125} = .688$

$z = \frac{.688 - .60}{\sqrt{\frac{(.60)\cdot(.40)}{125}}} = 2.0083$

using Table A

p value = .0446

Conclude: P-value = .0446 is less than $\alpha = .05 \rightarrow$ Reject H_0.

There is convincing evidence the true proportion of all teens who pass on the first try is not 60%.

(b) Find the corresponding confidence interval to see what the true proportion may be. Does your interval support your decision from the confidence interval above??

$ME = (1.96)\sqrt{\frac{(.688)(.312)}{125}}$

$\hat{p} = .688$

$z^* = 1.96$

$n = 125$

$.688 \pm .0812$

$= (.60678, .76922)$

Since .60 is not in the interval, we should have rejected, and we did.

7. IRHS makes a change that should improve student satisfaction with the parking situation *(we can dream)*. Before the change, 37% of the school's students approved of the parking that was provided. After the change, Mr. Munger surveys an SRS of 200 of the over 2900 students *(we can dream)* at the school. In all, 83 students say that they approve of the new parking arrangement. Mr. Munger cites this as evidence that the change was effective.

(a) Describe a Type I error and a Type II error in this setting, and explain the consequences of each.

Type I: Saying the true proportion of satisfied students is greater than 37%, but its not.

conseq. stop trying to improve, but its needed.

Type II: Saying the true satisfied proportion is not greater than 37%, but it is.

conseq. spending more \$, time, & effort & its not necessary.

(b) The test has a power of 0.75 to detect that $p = 0.45$. Explain what this means.

If the true proportion of satisfied students is .45, there is a 75% chance that we will find convincing evidence that $p > 37\%$ or $p > .37$

(c) Identify two ways to increase the power in part (b).

Increase power:

→ ↑ significance level (α)

→ ↑ sample size

8. Experiments on learning in animals sometimes measure how long it takes mice to find their way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. She measures how long each of 10 mice takes with a noise as stimulus. The appropriate hypotheses for the significance test are

(a) $H_0: \mu = 18$; $H_a: \cancel{\mu \neq 18}$.
(b) $H_0: \mu = 18$; $H_a: \cancel{\mu > 18}$.
(c) $\cancel{H_0: \mu < 18}$; $H_a: \cancel{\mu = 18}$.
(d) $H_0: \mu = 18$; $H_a: \mu < 18$.
(e) $\cancel{H_0: \bar{x} = 18}$; $H_a: \bar{x} < 18$.

Questions 9-11 refer to the following setting:
Members of the city council want to know if a majority of city residents supports a 1% increase in the sales tax to fund road repairs. To investigate, they survey a random sample of 300 city residents and use the results to test the following hypotheses: $H_0: p = 0.50$; $H_a: p > 0.50$
where p is the proportion of all city residents who support a 1% increase in the sales tax to fund road repairs.

9. A Type I error in the context of this study occurs if the city council

Reject H_0 (but shouldn't have)

(a) finds convincing evidence that a majority of residents supports the tax increase, when in reality there ~~isn't~~ convincing evidence that a majority supports the increase.
(b) finds convincing evidence that a majority of residents supports the tax increase, when in reality at most 50% of city residents support the increase.
(c) finds convincing evidence that a majority of residents supports the tax increase, when in reality more than 50% of city residents do support the increase.
(d) ~~does not find convincing~~ evidence that a majority of residents supports the tax increase, when in reality more ~~than 50% of city residents~~ do support the increase.
(e) ~~does not find convincing~~ evidence that a majority of residents supports the tax increase, when in reality at most ~~50% of city residents do~~ support the increase.

10. In the sample, $\hat{p}$ = 158/300 = 0.527. The resulting *P*-value is 0.18. What is the correct interpretation of this *P*-value?

(a) Only ~~18% of the city~~ residents support the tax increase.
(b) There is ~~an 18% chance~~ that the majority of residents supports the tax increase.
(c) Assuming that 50% of residents support the tax increase, there is an 18% probability that the sample proportion would be 0.527 or higher by chance alone.
(d) Assuming that ~~more than~~ 50% of residents support the tax increase, there is an 18% probability that the sample proportion would be 0.527 or higher by chance alone.
(e) Assuming that 50% of residents support the tax increase, there is an 18% chance that the ~~null hypothesis~~ is true by chance alone.

11. Based on the *P*-value in Question 11, which of the following would be the most appropriate conclusion?

(a) Because the *P*-value is large, we ~~reject H_0~~. We have convincing evidence that more than 50% of city residents support the tax increase.
(b) Because the *P*-value is large, we fail to reject H_0. We ~~have convincing~~ evidence that more than 50% of city residents support the tax increase.
(c) Because the *P*-value is large, we ~~reject H_0~~. We have convincing evidence that at most 50% of city residents support the tax increase.
(d) Because the *P*-value is large, we fail to reject H_0. We ~~have convincing~~ evidence that at most 50% of city residents support the tax increase.
(e) Because the *P*-value is large, we fail to reject H_0. We do not have convincing evidence that more than 50% of city residents support the tax increase.

12. Which of the following is ***not*** a condition for performing a significance test about a population proportion, p?

(a) The data should come from a random sample or randomized experiment.
(b) Both np_0 and $n(1 - p_0)$ should be at least 10.
(c) If you are sampling without replacement from a finite population then you should sample no more than 10% of the population.
(d) The population distribution should be approximately Normal, unless the sample size is large. → For means
(e) All of the above are conditions for performing a significance test about a population proportion.

13. After once again losing a football game to the archrival, a college's alumni association conducted a survey to see if alumni were in favor of firing the coach. An SRS of 100 alumni from the population of all living alumni was taken, and 64 of the alumni in the sample were in favor of firing the coach. Suppose you wish to see if a majority of living alumni are in favor of firing the coach. The appropriate test statistic is

(a) $z = \dfrac{0.64-0.5}{\sqrt{\frac{0.64(0.36)}{100}}}$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

(b) $\cancel{t = \dfrac{0.64-0.5}{\sqrt{\frac{0.64(0.36)}{100}}}}$

(c) $z = \dfrac{0.64-0.5}{\sqrt{\frac{0.5(0.5)}{100}}}$

(d) $z = \dfrac{0.64-0.5}{\sqrt{\frac{0.64(0.36)}{\cancel{64}}}}$

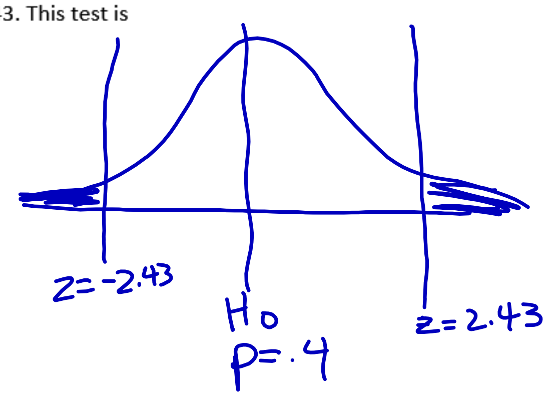
(e) $\cancel{z = \dfrac{0.5-0.64}{\sqrt{\frac{0.5(0.5)}{100}}}}$

14. Which of the following 95% confidence intervals would lead us to reject $H_0: p = 0.30$ in favor of the $H_a: p \neq 0.30$ at the 5% significance level?

(a) (0.19, 0.27)
(b) (0.24, 0.30)
(c) (0.27, 0.31)
(d) (0.29, 0.38)
(e) None of these

15. The z statistic for a test of $H_0: p = 0.4$ versus $H_a: p \neq 0.4$ is $z = 2.43$. This test is

(a) not significant at either $\alpha = 0.05$ or $\alpha = 0.01$.
(b) significant at $\alpha = 0.05$ but not at $\alpha = 0.01$.
(c) significant at $\alpha = 0.01$ but not at $\alpha = 0.05$.
(d) significant at both $\alpha = 0.05$ and $\alpha = 0.01$.
(e) inconclusive because we don't know the value of $\hat{p}$



normalcdf(LB: −1E99 UB: −2.43, μ: 0 σ = 1)

$\boxed{= .0075}$

$2(.0075) = .015$

P-val. < α → Reject H_0 (Data <u>is</u> Significant)

P-val > α → Fail to Reject (Data <u>is not</u> Significant)