6.1 Discrete Random Variables

pages 347–350

What is a random variable?

What is a probability distribution?

What is a discrete random variable?

Imagine selecting a U.S. high school student at random. Define the random variable X = number of languages spoken by the randomly selected student. The table below gives the probability distribution of X, based on a sample of students from the U.S. Census at School database.

Languages:	1	2	3	4	5
Probability:	0.630	0.295	0.065	0.008	0.002

(a) Show that the probability distribution for X is legitimate.
(b) Make a histogram of the probability distribution. Describe what you see.
(c) What is the probability that a randomly selected student speaks at least 3 languages? Interpret this probability.
(d) What is the probability that a randomly selected student speaks more than 3 languages? How is this different than (c)?

One wager players can make in Roulette is called a "corner bet." To make this bet, a player places his chips on the intersection of four numbered squares on the Roulette table. If one of these numbers comes up on the wheel and the player bet $1, the player gets his $1 back plus $8 more. Otherwise, the casino keeps the original $1 bet. If X = net gain from a single $1 corner bet, the possible outcomes are $x = -1$ or $x = 8$. Here is the probability distribution of X:

Value x	–$1	$8
Probability $p(x)$	34/38	4/38

If a player were to make this $1 bet over and over, what would be the player's average gain?

pages 350–352

How do you calculate the mean (expected value) of a discrete random variable? Is the formula on the formula sheet?

How do you interpret the mean (expected value) of a discrete random variable?

Calculate and interpret the mean number of languages in the example on the previous page.

Does the expected value of a random variable have to equal one of the possible values of the random variable? Should expected values be rounded?

6.1 Discrete and Continuous Random Variables
pages 352–354

Suppose that X is a discrete random variable with probability distribution to the right, and that μ_x is the mean of X.

Value:	x_1	x_2	x_3	…
Probability:	p_1	p_2	p_3	…

Variance of *X*:

Standard Deviation of *X*:

How do you interpret the standard deviation of a discrete random variable?

Calculate and interpret the standard deviation of X in the languages example.

How to calculate **mean and standard deviation** of a discrete random variable on the **calculator**:

- Enter x-values into list 1
- Enter probability values into list 2
- Calculate the 1-variable stats with list 1 in the list category
- Use list 2 as the frequency list
- Calculate

$\bar{x} = mean$, $S_x =$ standard deviation

In 2010, there were 1319 games played in the National Hockey League's regular season. Imagine selecting one of these games at random and then randomly selecting one of the two teams that played in the game. Define the random variable *X* = number of goals scored by a randomly selected team in a randomly selected game. The table below gives the probability distribution of *X*:

Goals:	0	1	2	3	4	5	6	7	8	9
Probability:	0.061	0.154	0.228	0.229	0.173	0.094	0.041	0.015	0.004	0.001

Sample: Compute the mean and standard deviation of the random variable X and interpret this value in context.

In an experiment on the behavior of young children, each subject is placed in an area with five toys. Past experiments have shown that the probability distribution of the number X of toys played with by a randomly selected subject is as follows:

X toys	0	1	2	3	4	5
Probability	0.03	0.16	0.30	0.23	0.17	0.11

Sample: Calculate the mean and standard deviation of the random variable X and interpret this value in context.

What is a continuous random variable?

Is it possible to have a shoe size = 8? Is it possible to have a foot length = 8 inches?

How many possible foot lengths are there? How can we graph the distribution of foot length?

How do we find probabilities for continuous random variables?

For a continuous random variable X, how is $P(X < \text{a})$ related to $P(X \leq \text{a})$?

The weights of three-year-old females closely follow a Normal distribution with a mean of $\mu = 30.7$ pounds and a standard deviation of $\sigma = 3.6$ pounds. Randomly choose one three-year-old female and call her weight X.

(a) Find the probability that the randomly selected three-year-old female weighs at least 30 pounds.

(b) Find the probability that a randomly selected three-year-old female weighs between 25 and 35 pounds.

(c) If $P(X < k) = 0.8$, find the value of k.

6.2 Transforming RVs

pages 363–369

El Dorado Community College considers a student to be full-time if he or she is taking between 12 and 18 units. The number of units X that a randomly selected El Dorado Community College full-time student is taking in the fall semester has the following distribution.

Number of Units (X)	12	13	14	15	16	17	18
Probability	0.25	0.10	0.05	0.30	0.10	0.05	0.15

Draw a probability histogram. Then, calculate and interpret the mean and standard deviation of X.

At El Dorado Community College, the tuition for full-time students is \$50 per unit. So, if T = tuition charge for a randomly selected full-time student, $T = 50X$. Here's the probability distribution for T:

Tuition Charge (T)	600	650	700	750	800	850	900
Probability	0.25	0.10	0.05	0.30	0.10	0.05	0.15

Calculate the mean and standard deviation of T.

What is the effect of multiplying or dividing a random variable by a constant?

In addition to tuition charges, each full-time student at El Dorado Community College is assessed student fees of $100 per semester. If C = overall cost for a randomly selected full-time student, $C = 100 + T$. Here is the probability distribution for C:

Overall Cost (C)	700	750	800	850	900	950	1000
Probability	0.25	0.10	0.05	0.30	0.10	0.05	0.15

Calculate the mean and standard deviation of C.

What is the effect of adding (or subtracting) a constant to a random variable?

What is a linear transformation? How does a linear transformation affect the mean and standard deviation of a random variable?

In a large introductory statistics class, the distribution of X = raw scores on a test was approximately Normally distributed with a mean of 17.2 and a standard deviation of 3.8. The professor decides to scale the scores by multiplying the raw scores by 4 and adding 10.

(a) Define the variable Y to be the scaled score of a randomly selected student from this class. Find the mean and standard deviation of Y.

(b) What is the probability that a randomly selected student has a scaled test score of greater than 90?

6.2 Combining Random Variables

pages 369–381

The only way to determine the probability for any value of T is if X and Y are ____________________ random variables.

Definition: If knowing whether any event involving X alone has occurred tells us __________________ about the occurrence of any event involving Y alone, and vice versa, then X and Y are independent random variables.

- Probability models often ___________________ independence when the random variables describe outcomes that appear ___________________to each other.
- You should always ask whether the assumption of independence seems ______________________.

Rule: Mean of the Sum of Random Variables

For any two random variables X and Y, if T = X + Y, then the expected value of T is:

$E_T = \mu_T =$

In general, the mean of the sum of several random variables is the __________________________.

Rule: Variance of the Sum of Random Variables

When we add two independent random variables, their ______________________ add.
Standard deviations _____________________ add.

For any two independent random variables X and Y, T = X + Y, then the variance (Var) of T is:

$Var_T =$

and since variance is standard deviation squared

$\sigma^2_T =$

so we can find standard deviation by

$\sigma_T =$

In general, the variance of the sum of several independent random variables is the ____________ of their variances.

Suppose that a certain variety of apples have weights that are approximately Normally distributed with a mean of 9 ounces and a standard deviation of 1.5 ounces. If bags of apples are filled by randomly selecting 12 apples, what is the probability that the sum of the 12 apples is less than 100 ounces?

Let B = the amount spent on books in the fall semester for a randomly selected full-time student at El Dorado Community College. Suppose that $\mu_B = 153$ and $\sigma_B = 32$. Recall from earlier that C = overall cost for tuition and fees for a randomly selected full-time student at El Dorado Community College and μ_C = 832.50 and σ_C = 103. Find the mean and standard deviation of the cost of tuition, fees and books ($C + B$) for a randomly selected full-time student at El Dorado Community College. What is the shape of the distribution?