

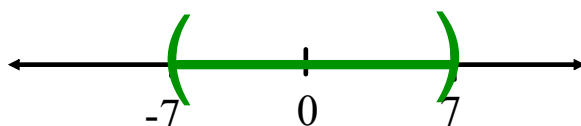
Absolute value inequalities of the form:  $|x| < a$

All the numbers whose distance from zero is less than a.

AND  $|m| < 7$

$-7 < m < 7$

What are all the numbers that are less than 7 units from zero?



<p><math> x + 1  &lt; 4</math></p> <p> <math>x + 1 &lt; 4</math>  <math>-1 \quad -1</math>  <math>\hline</math>  <math>x &lt; 3</math> </p> <p> <math>x + 1 &gt; -4</math>  <math>-1 \quad -1</math>  <math>\hline</math>  <math>x &gt; -5</math> </p> <p> <math>-4 &lt; x + 1 &lt; 4</math>  <math>-1 \quad -1 \quad -1 \quad -1</math>  <math>\hline</math>  <math>-5 &lt; x &lt; 3</math> </p>	<p><math> m + 1  &lt; -5</math></p> <p> <math>m + 1 &lt; -5</math>    <math>m + 1 &gt; 5</math>  <math>-1 \quad -1</math>    <math>-1 \quad -1</math>  <math>\hline</math>    <math>\hline</math>  <math>m &lt; -6</math>    <math>m &gt; 4</math> </p> <p><u>No Solution</u></p>
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*① get abs. val by itself*

$$|x - 2| + 1 \leq 4$$

$$\begin{array}{r} -1 \quad -1 \\ \hline |x - 2| \leq 3 \end{array}$$

$x - 2 \leq 3$   
 $\begin{array}{r} +2 \quad +2 \\ \hline x \leq 5 \end{array}$

$x - 2 \geq -3$   
 $\begin{array}{r} +2 \quad +2 \\ \hline x \geq -1 \end{array}$

$[ -1, 5 ]$

$$\begin{array}{r} -3 \leq x - 2 \leq 3 \\ +2 \quad +2 \quad +2 \\ \hline -1 \leq x \leq 5 \end{array}$$

$$|n - 2| \leq -6$$

$$\begin{array}{r} 6 \leq n - 2 \leq -6 \\ +2 \quad +2 \quad +2 \\ \hline 8 \leq n \leq -4 \end{array}$$

$n \geq 8$

No Solution

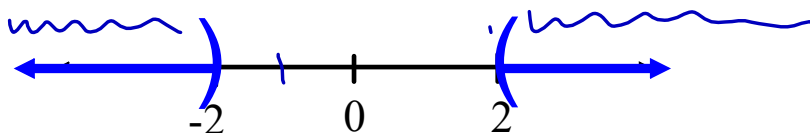
Absolute value inequalities of the form:  $|x| > a$

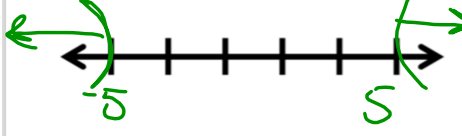

All the numbers whose distance from zero is greater than a.

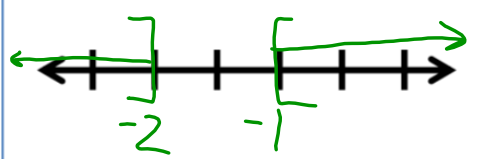
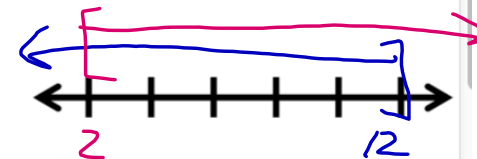
$$|x| > 2$$

$$x < -2 \text{ or } x > 2$$

What are all the numbers that are greater than 2 units from zero?



$\frac{3 2x }{3} > \frac{30}{3}$ $ 2x  > 10$ $\frac{2x}{2} > \frac{10}{2} \quad \frac{2x}{2} < \frac{-10}{2}$ $x > 5 \quad x < -5$ $(-\infty, -5) \cup (5, \infty)$ 	$ 4x - 2  > -8$ $\begin{array}{l} 4x - 2 > -8 \\ +2 \quad +2 \\ \hline 4x > -6 \\ \frac{4x}{4} > \frac{-6}{4} \\ x > -\frac{3}{2} \end{array}$ $\begin{array}{l} 4x - 2 < 8 \\ +2 \quad +2 \\ \hline 4x < 10 \\ \frac{4x}{4} < \frac{10}{4} \\ x < \frac{5}{2} \end{array}$ $(-\infty, \infty)$ 
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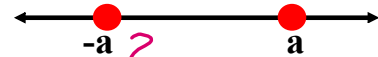
$ 2x + 3  \geq 1$ $\frac{2x + 3 \geq 1}{-3 \quad -3} \quad \frac{2x + 3 \leq -1}{-3 \quad -3}$ $\frac{2x \geq -2}{2} \quad \frac{2x \leq -4}{2}$ $x \geq -1 \quad x \leq -2$ $(-\infty, -2] \cup [-1, \infty)$ 	$ x - 7  \geq -5$ $\frac{x - 7 \geq -5}{+7 \quad +7} \quad \frac{x - 7 \leq 5}{+7 \quad +7}$ $x \geq 2 \quad x \leq 12$ $(-\infty, \infty)$ 
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Solving Absolute Value Equations and Inequalities with  $a > 0$ .

Algebraic Solutions

Solution Graph

$|X| = a$  is equivalent to  $X = a$  or  $X = -a$ .



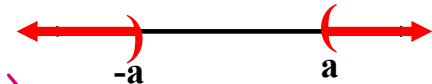
$\{-a, a\}$

$|X| < a$  is equivalent to  $-a < X < a$ .



$(-a, a)$

$|X| > a$  is equivalent to  $X < -a$  or  $X > a$ .



$(-\infty, -a) \cup (a, \infty)$

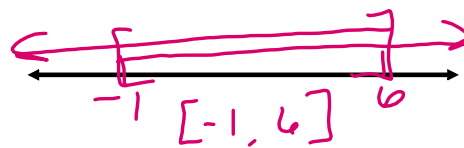
and

OR

OR looks like OARS

Before we use an absolute value inequality property we must first isolate the absolute value expression.

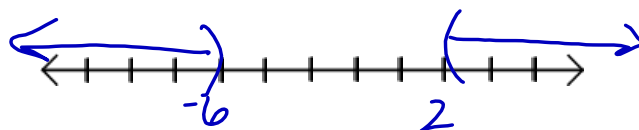
$$\begin{aligned}
 |2x - 5| + 2 &\leq 9 \\
 &\quad -2 \quad -2 \\
 \hline
 |2x - 5| &\leq 7 \\
 -7 &\leq 2x - 5 \leq 7 \\
 +5 &\quad +5 \quad +5 \\
 \hline
 -\frac{2}{2} &\leq \frac{2x}{2} \leq \frac{12}{2} \\
 -1 &\leq x \leq 6 \\
 &\quad \underbrace{\hspace{1cm}}_{x \geq -1}
 \end{aligned}$$



Remember: You cannot distribute into an absolute value, divide by the value instead.

$$5 - 3|v + 2| < -7$$

$$\begin{array}{r} -5 \qquad -5 \\ \hline -3|v+2| < -12 \\ \hline \downarrow -3 \quad \downarrow -3 \end{array}$$



$$(-\infty, -6) \cup (2, \infty)$$

$$|v+2| > 4$$

$$\begin{array}{r} v+2 > 4 \\ -2 \quad -2 \\ \hline v > 2 \end{array} \qquad \begin{array}{r} v+2 < -4 \\ -2 \quad -2 \\ \hline v < -6 \end{array}$$

$$5 - 3|-7+2| < -7$$

$$5 - 3|-5| < -7$$

$$5 - 15 < -7$$

$$-10 < -7 \quad \checkmark$$

Practice:

Absolute Value Inequalities