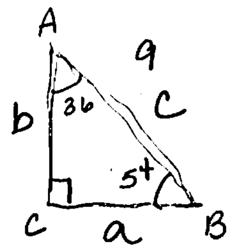
Chapter 9 Practice Test

Solve $\triangle ABC$ by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

1. $A = 36°, c = 9$

SOLUTION:

Find the measure of the third angle.



$$m\angle B = 180° - (36° + 90°)$$
$$= 54$$

Use the Law of Sines.

Find the value of a.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \text{ [Law of Sines]}$$
$$\frac{\sin 36°}{a} = \frac{\sin 90°}{9}$$
$$a = \frac{9 \sin 36}{\sin 90}$$
$$\approx 5.3$$

$\sin 36° = \frac{a}{9}$

$9 \sin 36 = a$

$5.3 = a$

We used trig ratios not Law of Sines

Answers are the same

Find the value of b.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \text{ [Law of Sines]}$$
$$\frac{\sin 36}{5.3} = \frac{\sin 54}{b}$$
$$b = \frac{5.3 \sin 54}{\sin 36}$$
$$\approx 7.3$$

ANSWER:

$B = 54°, a = 5.3, b = 7.3$

2. $a = 12, A = 58°$

SOLUTION:

Find the measure of the third angle.

$$m\angle B = 180° - (58° + 90°)$$
$$= 32$$

Use the Law of Sines to find side length c.

$$\frac{\sin A}{a} = \frac{\sin C}{c} \text{ [Law of Sines]}$$
$$\frac{\sin 58°}{12} = \frac{\sin 90°}{c}$$
$$c = \frac{12 \sin 90°}{\sin 58}$$
$$c = \frac{12(1)}{0.848}$$
$$\approx 14.2$$

Find the value of b.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \text{ [Law of Sines]}$$
$$\frac{\sin 58°}{12} = \frac{\sin 32°}{b}$$
$$b = \frac{12 \sin 32°}{\sin 58}$$
$$b = \frac{12(0.53)}{0.848}$$
$$\approx 7.5$$

ANSWER:

$B = 32°, c = 14.2, b = 7.5$

3. $B = 85°, b = 8$

SOLUTION:

Find the measure of the third angle.

$$m\angle A = 180^\circ - (85^\circ + 90^\circ)$$
$$= 5^\circ$$

Use the Law of Sines to find side length c.

$$\frac{\sin B}{b} = \frac{\sin C}{c} \text{ [Law of Sines]}$$
$$\frac{\sin 85^\circ}{8} = \frac{\sin 90^\circ}{c}$$
$$c = \frac{8 \sin 90^\circ}{\sin 85^\circ}$$
$$\approx 8.0$$

Find the value of b.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \text{ [Law of Sines]}$$
$$\frac{\sin 5^\circ}{a} = \frac{\sin 85^\circ}{8}$$
$$a = \frac{8 \sin 5^\circ}{\sin 85^\circ}$$
$$\approx 0.7$$

ANSWER:

$A = 5°, c = 8.0, a = 0.7$

4. $a = 9, c = 12$

SOLUTION:

$$\frac{\sin A}{a} = \frac{\sin C}{c} \text{ [Law of Sines]}$$
$$\frac{\sin A}{9} = \frac{\sin 90^\circ}{12}$$
$$A = \sin^{-1}\left(\frac{9}{12}\right)$$
$$A \approx 49^\circ$$

Find the measure of the third angle.

$$m\angle B \approx 180^\circ - (49^\circ + 90^\circ) \text{ or } 41^\circ$$

Use the Law of Sines to find the length b.

$$\frac{\sin C}{c} = \frac{\sin B}{b} \text{ [Law of Sines]}$$
$$\frac{\sin 90^\circ}{12} = \frac{\sin 41^\circ}{b}$$
$$b = \frac{12 \sin 41^\circ}{\sin 90^\circ}$$
$$\approx 7.9$$

ANSWER:

$b = 7.9, B = 41°, A = 49°$

Rewrite each degree measure in radians and each radian measure in degrees.

5. 325°

SOLUTION:

$$325^\circ = 325^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$$
$$= \frac{325\pi}{180} \text{ radians}$$
$$= \frac{65\pi}{36} \text{ radians}$$

ANSWER:

$$\frac{65\pi}{36}$$

6. −175°

SOLUTION:

$$\begin{aligned}-175^\circ &= -175^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} \\ &= -\frac{175\pi}{180} \text{ radians} \\ &= -\frac{35\pi}{36} \text{ radians}\end{aligned}$$

ANSWER:

$-\frac{35\pi}{36}$

7. $\frac{9\pi}{4}$

SOLUTION:

$$\begin{aligned}\frac{9\pi}{4} &= \frac{9\cancel{\pi}}{4} \cancel{\text{radians}} \cdot \frac{180^\circ}{\cancel{\pi} \cancel{\text{radians}}} \\ &= 405^\circ\end{aligned}$$

ANSWER:

405°

8. $-\frac{5\pi}{6}$

SOLUTION:

$$\begin{aligned}-\frac{5\pi}{6} &= -\frac{5\cancel{\pi}}{6} \cancel{\text{radians}} \cdot \frac{180^\circ}{\cancel{\pi} \cancel{\text{radians}}} \\ &= -150^\circ\end{aligned}$$

ANSWER:

−150°

Find the exact value of each function. Write angle measures in degrees.

9. cos (−90°)

SOLUTION:

$$\begin{aligned}-90^\circ &= 360 - 90 \\ &= 270^\circ\end{aligned}$$

Since the angle 270° is a quadrant angle, the coordinates of the point on its terminal side is (0, −*y*).

Find the value of *r*.

$$\begin{aligned}r &= \sqrt{a^2 + b^2} \\ &= \sqrt{0^2 + (-y)^2}\end{aligned}$$

$$r = y$$

$$\begin{aligned}\cos(-90^\circ) &= \cos 90^\circ \\ &= \frac{x}{r} \\ &= \frac{0}{y} \\ &= 0\end{aligned}$$

ANSWER:

0

10. $\sin 585°$

SOLUTION:

$$\begin{aligned}\sin 585° &= \sin(360° + 225°)\\ &= \sin 225°\end{aligned}$$

The terminal side of $225°$ lies in Quadrant III.

Find the measure of the reference angle.

$$\begin{aligned}\theta' &= \theta - 180°\\ &= 225° - 180°\\ &= 45°\end{aligned}$$

The sine function is negative in quadrant III.

$$\begin{aligned}\sin(585°) &= -\sin 45°\\ &= -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

ANSWER:

$-\frac{\sqrt{2}}{2}$

11. $\cot \frac{4\pi}{3}$

SOLUTION:

The terminal side of $\frac{4\pi}{3}$ lies in Quadrant III.

Find the measure of the reference angle.

$$\begin{aligned}\theta' &= \theta - \pi\\ &= \frac{4\pi}{3} - \pi\\ &= \frac{\pi}{3}\end{aligned}$$

The cotangent function is positive in quadrant III.

$$\begin{aligned}\cot\left(\frac{4\pi}{3}\right) &= \cot\left(\frac{\pi}{3}\right)\\ &= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}\\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

ANSWER:

$\frac{\sqrt{3}}{3}$

12. $\sec\left(-\frac{9\pi}{4}\right)$

SOLUTION:

$$\begin{aligned}\sec\left(-\frac{9\pi}{4}\right) &= \sec\left(\frac{9\pi}{4}\right)\\ &= \sec\left(2\pi + \frac{\pi}{4}\right)\\ &= \sec\left(\frac{\pi}{4}\right)\\ &= \sqrt{2}\end{aligned}$$

ANSWER:

$\sqrt{2}$

13. The terminal side of angle θ in standard position intersects the unit circle at point $P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Find $\cos\theta$ and $\sin\theta$.

SOLUTION:

$$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = P(\cos\theta, \sin\theta)$$

Therefore, $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$.

ANSWER:

$\cos\theta = \frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$

14. **MULTIPLE CHOICE** What angle has a tangent and sine that are both negative?

A 65°
B 120°
C 120°
D 310°

SOLUTION:

In the fourth quadrant both the sine and the tangent are negative. The angle 310° lies in the fourth quadrant. Therefore, option D is the correct answer.

ANSWER:

D

15. **NAVIGATION** Airplanes and ships measure distance in nautical miles. The formula 1 nautical mile = 6077 − 31 cos 2θ feet, where θ is the latitude in degrees, can be used to find the approximate length of a nautical mile at a certain latitude. Find the length of a nautical mile when the latitude is 120°.

SOLUTION:

Substitute 120° for θ in the given equation and evaluate.

$$\begin{aligned}\text{length} &= 6077 - 31\cos 2(120^\circ)\\ &= 6092.5\end{aligned}$$

Therefore, the length of a nautical mile when the latitude is 120° is 6092.5 feet.

ANSWER:

6092.5 ft

Find the amplitude and period of each function. Then graph the function.

16. $y = 2\sin 3\theta$

SOLUTION:

Given $a = 2$ and $b = 3$.

Amplitude:

$$\begin{aligned}|a| &= |2|\\ &= 2\end{aligned}$$

16- y = 2 sin θ

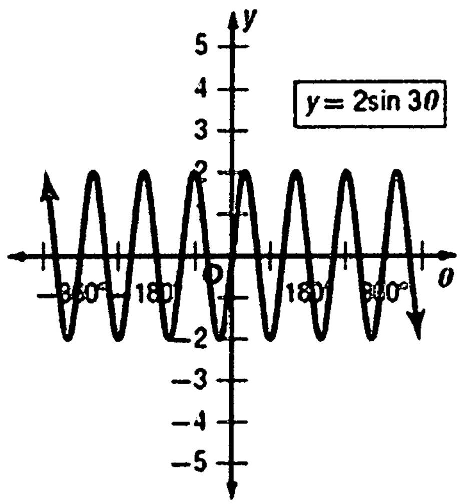
Period:

$\frac{360^\circ}{|b|} = \frac{360}{|3|}$

$= 120$

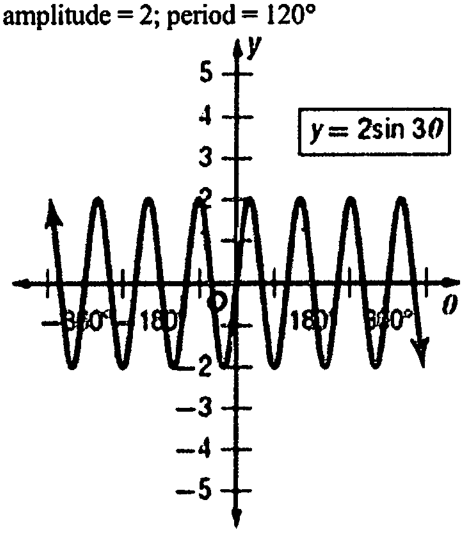
a=2
b=3
Amplitude = 2

Graph the function.



ANSWER:

amplitude = 2; period = 120°



17. $y = \frac{1}{2}\cos 2\theta$

SOLUTION:

Given $a = \frac{1}{2}$ and $b = 2$.

Amplitude:

$|a| = \left|\frac{1}{2}\right|$

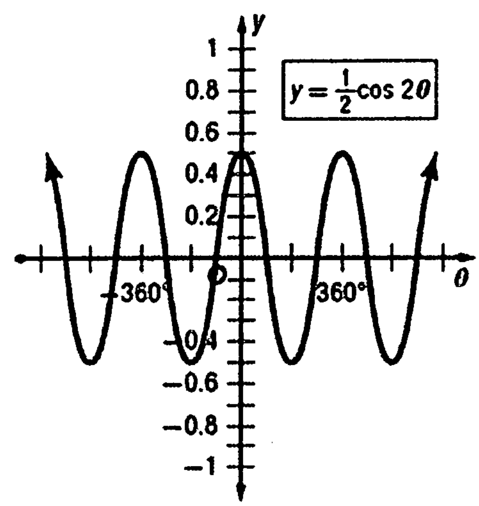
$= \frac{1}{2}$

Period:

$\frac{360^\circ}{|b|} = \frac{360}{|2|}$

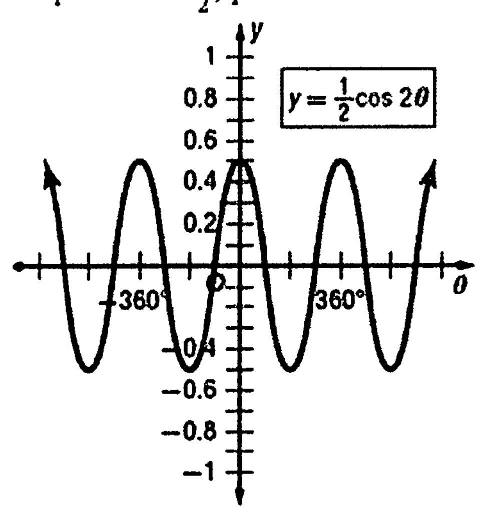
$= 180^\circ$

Graph the function.



ANSWER:

amplitude = $\frac{1}{2}$; period = 180°



18. **MULTIPLE CHOICE** What is the period of the function $y = 3 \cot \theta$?

A 120°
B 180°
C 360°
D 1080°

SOLUTION:

Given $b = 1$.

Period:

$$\frac{180^\circ}{|b|} = \frac{180^\circ}{|1|}$$
$$= 180^\circ$$

The option B is the correct option.

ANSWER:

B

Write an equation for each translation.

19. $y = \sin x$, 6 units to the left and 4 units down

SOLUTION:

$y = \sin(x + 6) - 4$

ANSWER:

$y = \sin(x + 6) - 4$

20. $y = \tan x$, $\frac{\pi}{2}$ units to the right and 1 unit up

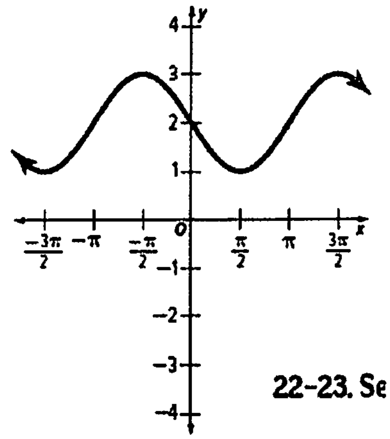
SOLUTION:

$y = \tan\left(x - \frac{\pi}{2}\right) + 1$

ANSWER:

$y = \tan\left(x - \frac{\pi}{2}\right) + 1$

21. Write an equation for the graph using the given trigonometric function.



SOLUTION:

Sample answer: $y = \cos\left(\theta + \frac{\pi}{2}\right) + 2$

ANSWER:

Sample answer: $y = \cos\left(\theta + \frac{\pi}{2}\right) + 2$

State the amplitude, period, and phase shift for each function. Then graph the function.

22. $y = \cos(\theta + 180)$

SOLUTION:

Given $a = 1$, $b = 1$ and $h = -180°$.

Amplitude:

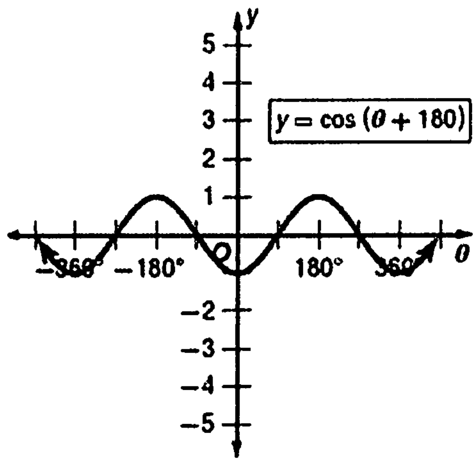
$$|a| = |1|$$
$$= 1$$

Period:

$$\frac{360^\circ}{|b|} = \frac{360^\circ}{|1|}$$
$$= 360^\circ$$

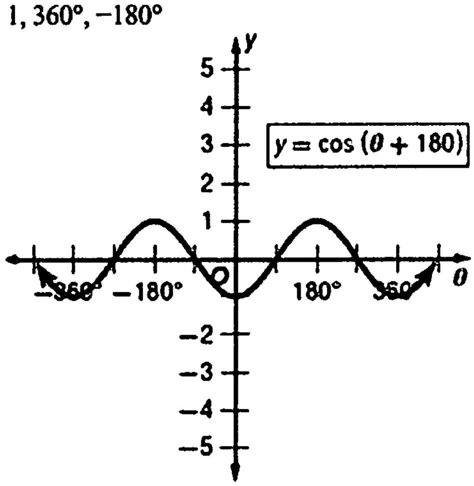
Phase shift: $h = -180^\circ$

Draw the graph of $y = \cos\theta$. Then shift the graph 180° to the left.



ANSWER:

1, 360°, −180°



23. $y = \frac{1}{2}\tan\left(\theta - \frac{\pi}{2}\right)$

SOLUTION:

Given $a = \frac{1}{2}$, $b = 1$ and $h = \frac{\pi}{2}$.

Amplitude:
Amplitude does not exist.

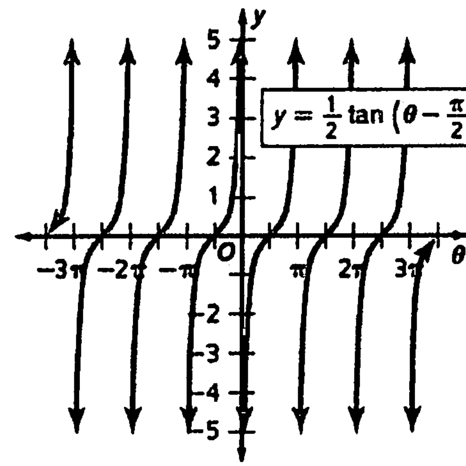
Period:

$\frac{\pi}{|b|} = \frac{\pi}{|1|}$

$= \pi$

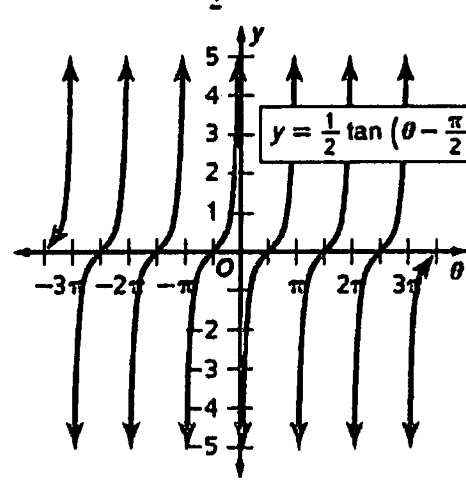
Phase shift: $h = \frac{\pi}{2}$

Draw the graph of $y = \frac{1}{2}\tan\theta$. Then shift the graph $\frac{\pi}{2}$ units to the right.

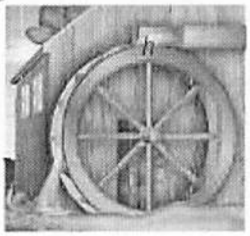


ANSWER:

does not exist, π, $\frac{\pi}{2}$



24. **WHEELS** A water wheel has a diameter of 20 feet. It makes one complete revolution in 45 seconds. Let the height at the top of the wheel represent the height at time 0. Write an equation for the height of point h in the diagram below as a function of time t. Then graph the function.



SOLUTION:

The maximum and the minimum height is 20ft and 0 ft.

As the midline lies halfway between the maximum and the minimum values, the equation of the midline is $y = \frac{20-0}{2} = 10$.

Therefore the vertical shift $k = 10$.

Amplitude:

$|a| = |20 - 10|$
$= 10$

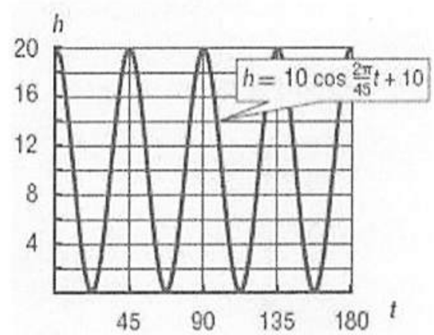
Period:

Since the wheel makes one complete revolution in 45 seconds, the period is 45 seconds.

$45 = \frac{2\pi}{|b|}$
$|b| = \frac{2\pi}{45}$
$b = \pm\frac{2\pi}{45}$

Substitute 10 for a, $\frac{2\pi}{45}$ for b, 0 for h, and 10 for k in

$h = a \cos b(t-h) + k$
$= 10 \cos \frac{2\pi}{45}(t-0) + 10$
$= 10 \cos \frac{2\pi}{45}t + 10$



ANSWER:

$h = 10\cos\frac{2\pi}{45}t + 10$