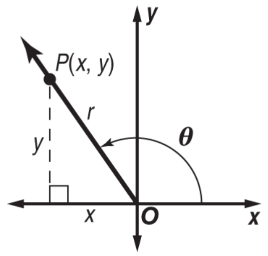
NAME ____ DATE _____________________________ PERIOD _____________

Trigonometric Functions of General Angles

Trigonometric Functions for General Angles

Trigonometric Functions, θ in Standard Position	Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. By the Pythagorean Theorem, the distance r from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.
	$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}, x \neq 0$ $\csc\theta = \frac{r}{y}, y \neq 0$ $\sec\theta = \frac{r}{x}, x \neq 0$ $\cot\theta = \frac{x}{y}, y \neq 0$

Example: Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-5, 5\sqrt{2})$.

You know that $x = -5$ and $y = 5\sqrt{2}$. You need to find r.

$r = \sqrt{x^2 + y^2}$ Pythagorean Theorem

$= \sqrt{(-5)^2 + (5\sqrt{2})^2}$ Replace x with –5 and y with $5\sqrt{2}$.

$= \sqrt{75}$ or $5\sqrt{3}$

Now use $x = -5$, $y = 5\sqrt{2}$, and $r = 5\sqrt{3}$ to write the six trigonometric ratios.

$\sin\theta = \frac{y}{r} = \frac{5\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{6}}{3}$ $\cos\theta = \frac{x}{r} = \frac{-5}{5\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\tan\theta = \frac{y}{x} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{5\sqrt{3}}{5\sqrt{2}} = \frac{\sqrt{6}}{2}$ $\sec\theta = \frac{r}{x} = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$ $\cot\theta = \frac{x}{y} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$

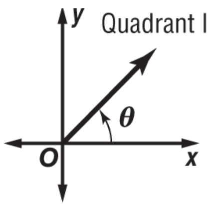
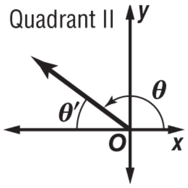
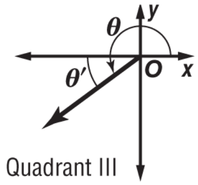
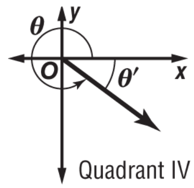
Exercises

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

1. (8, 4)   **2.** (4, 4)

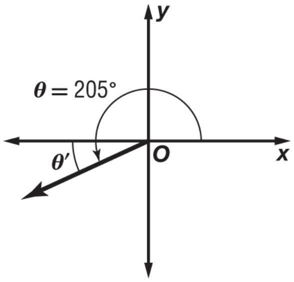
3. (0, 4)   **4.** (6, 2)

Trigonometric Functions with Reference Angles If θ is a nonquadrantal angle in standard position, its **reference angle** θ' is defined as the acute angle formed by the terminal side of θ and the x-axis.

Reference Angle Rule				
	 $\theta' = \theta$	 $\theta' = 180° - \theta$ $(\theta' = \pi - \theta)$	 $\theta' = \theta - 180°$ $(\theta' = \theta - \pi)$	 $\theta' = 360° - \theta$ $(\theta' = 2\pi - \theta)$

Example 1: Sketch an angle of measure 205°. Then find its reference angle.

Because the terminal side of 205° lies in Quadrant III, the reference angle θ' is 205° – 180° or 25°.



Example 2: Use a reference angle to find the exact value of cos $\frac{3\pi}{4}$.

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.

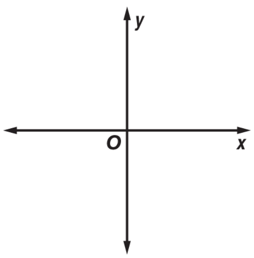
The cosine function is negative in Quadrant II.

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

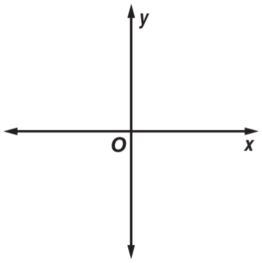
Exercises

Sketch each angle. Then find its reference angle.

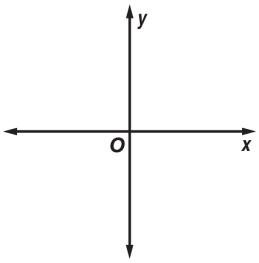
1. 155°



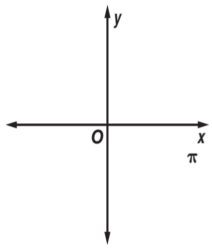
2. 230°



3. $\frac{4\pi}{3}$



4. $\frac{11\pi}{6}$



Find the exact value of each trigonometric expression.

5. tan 330°

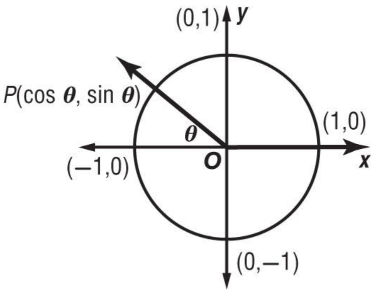
6. cos $\frac{11\pi}{4}$

7. cot 30°

8. csc $\frac{\pi}{4}$

Circular and Periodic Functions

Circular Functions

Definition of Sine and Cosine	If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$. 

Example: The terminal side of angle θ in standard position intersects the unit circle at $P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$. Find $\cos \theta$ and $\sin \theta$.

$P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right) = P(\cos \theta, \sin \theta)$, so $\cos \theta = -\frac{5}{6}$ and $\sin \theta = \frac{\sqrt{11}}{6}$.

Exercises

The terminal side of angle θ in standard position intersects the unit circle at each point P. Find $\cos \theta$ and $\sin \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. $P(0, -1)$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

7. P is on the terminal side of $\theta = 45°$.

8. P is on the terminal side of $\theta = 120°$.

9. P is on the terminal side of $\theta = 240°$.

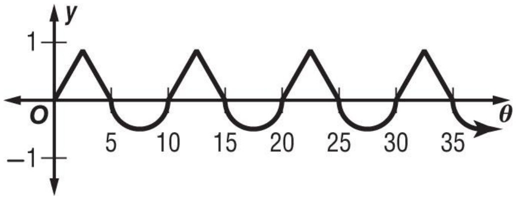
10. P is on the terminal side of $\theta = 330°$.

Periodic Functions

A **periodic function** has *y*-values that repeat at regular intervals. One complete pattern is called a **cycle**, and the horizontal length of one cycle is called a **period**.

The sine and cosine functions are periodic; each has a period of 360° or 2π radians.

Example 1: Determine the period of the function.



The pattern of the function repeats every 10 units, so the period of the function is 10.

Example 2: Find the exact value of each expression.

a. sin 855°

$\sin 855° = \sin(135° + 720°)$

$= \sin 135°$ or $\frac{\sqrt{2}}{2}$

b. $\cos\left(\frac{31\pi}{6}\right)$

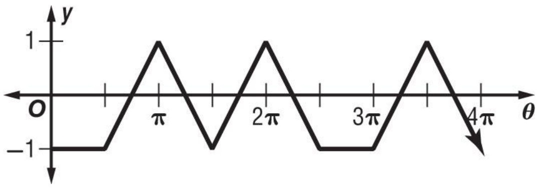
$\cos\left(\frac{31\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + 4\pi\right)$

$= \cos\frac{7\pi}{6}$ or $-\frac{\sqrt{3}}{2}$

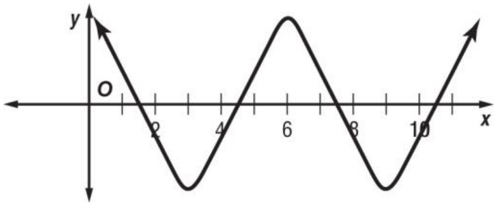
Exercises

Determine the period of each function.

1.



2.



Find the exact value of each expression.

3. sin (–510°)

4. sin 495°

5. $\cos\left(-\frac{5\pi}{2}\right)$

6. $\sin\left(\frac{5\pi}{3}\right)$

7. $\cos\left(\frac{11\pi}{4}\right)$

8. $\sin\left(-\frac{3\pi}{4}\right)$