

Probability

If event “E” has $n(E)$ equally likely outcomes, and its sample space “S” has $n(S)$ equally likely outcomes, then

the **PROBABILITY**

of event “E” is given by:

$$0 \leq P(E) \leq 1$$

If an event is **IMPOSSIBLE**,
It has a probability of **ZERO**

If an event is **CERTAIN**,
It has a probability of **ONE**

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{\text{Favorable}}{\text{Total}}$$



Geometric Probability

DART BOARD A dart is tossed and hits the dart board shown. The dart is equally likely to land on any point on the dart board. Find the probability that the dart lands in the red region.

SOLUTION

Find the ratio of the area of the red region to the area of the dart board.

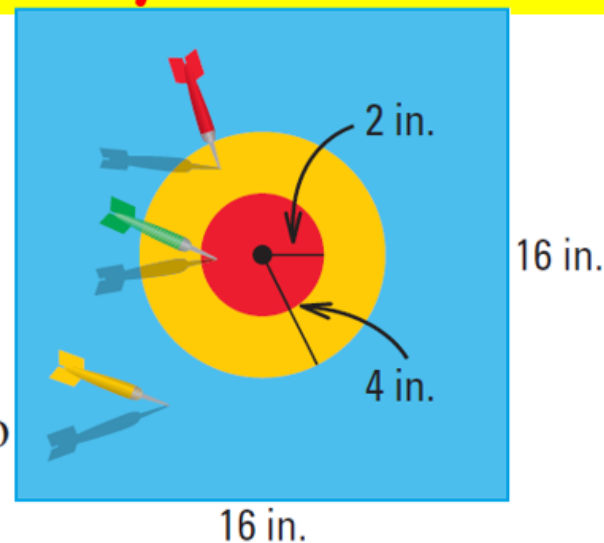
$P(\text{Dart lands in red region})$

$$P(E) = \frac{\text{Favorable}}{\text{Total}}$$

$$= \frac{\pi(2^2)}{16^2}$$

$$= \frac{4\pi}{256}$$

$$\approx 0.05$$



► The probability that the dart lands in the red region is about 0.05, or 5%.

COMBINATIONS

Finding the probability of a combination where order does not matter.

The number of combinations of n distinct objects taken r at a time is indicated by ${}_n C_r$, and is given by the formula:

$${}_n C_r = \frac{{}_n P_r}{r!} = \frac{n!}{(n-r)! (r)!}$$

$n!$ is a factorial

How a factorial works:

$$\frac{52!}{47!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \cancel{47} \cdot \cancel{46} \cdot \dots \cdot \cancel{2} \cdot \cancel{1}}{\cancel{47} \cdot \cancel{46} \cdot \dots \cdot \cancel{2} \cdot \cancel{1}}$$

8 Nuclear Physicists and 5 Nuclear Chemists have all applied to be on the next NASA team of 5 lead scientists. What is the probability that 3 Physicists and 2 Chemists are chosen?

What is the total number of things that could happen (the sample space)?

This is a combination because order doesn't matter.

$${}_{13} C_5 = \frac{13!}{(13-5)!5!} = 1287$$

First we need to determine how many groups of 3 Physicists can be formed....

$${}_8 C_3 = \frac{8!}{(8-3)!3!} = 56$$

Next we need to determine how many groups of 2 Chemists can be formed....

$${}_5 C_2 = \frac{5!}{(5-2)!2!} = 10$$

To determine the probability we calculate:

$$\frac{{}_8 C_3 \cdot {}_5 C_2}{{}_{13} C_5} = \frac{56 \cdot 10}{1287} = \frac{560}{1287}$$

Probability of Independent & Dependent events

Event	Rule
Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed	When two events, A & B, are dependent, the probability of both occurring is: $P(A \text{ and } B) = P(A) \times P(B A)$

Example A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

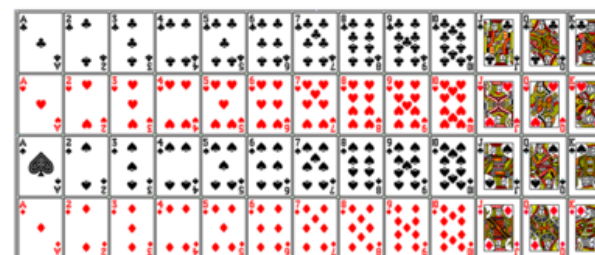
$$P(\text{queen on first pick}) = \frac{4}{52}$$

$$P(\text{jack on 2nd pick given queen on 1st pick}) = \frac{4}{51}$$

$$P(\text{queen and jack}) = \frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$$

AND

Means multiply probabilities



Probability

How about:
What is the probability that at least one person is 25 years old?

Example:

In 2004, 65% of the population was 25 or older. If 10 people are chosen at random, what is the probability that all 10 were 25 years or older?

Since there are 10 people chosen at random

$$.65 \cdot .65 \cdot .65 \cdot .65 \cdot .65 \cdot .65 \cdot .65 \cdot .65 \cdot .65 \cdot .65$$

$$(.65)^{10} = 0.01346$$

What is the probability that NOT ALL of the 10 were over 25 years old?

$$P(A^c) = 1 - 0.01346 = 0.98654$$

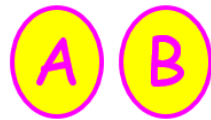
The Probability is 0.01346



Probability of Inclusive & Mutually exclusive events

Key Concepts

Mutually Exclusive: Two events A and B are Mutually Exclusive if A and B have no outcomes in common



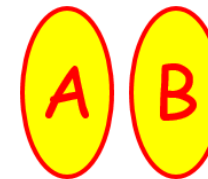
← Mutually Exclusive

NOT Mutually Exclusive →



If two events, A and B , are mutually exclusive, then the probability of A or B occurring is:

$$P(A \text{ OR } B) = P(A) + P(B)$$



If two events, A and B , are NOT mutually exclusive, then the probability of A or B occurring is:

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ and } B)$$

