Base e: Natural Logarithms

Main Ideas/Questions	Notes/Examples
What is "e"?	• e is an irrational number with an approximate value of 2.71828.... • e often occurs as the **base of exponential and logarithmic functions** that describe **real-world scenarios**.
Base "e" Exponential Functions	• Exponential functions with base e are called natural base exponential functions. • Example: $f(x) = e^x$
Base "e" Logarithmic Functions	• Logarithmic functions with base e are called natural log. • Example: $f(x) = \log_e x$. This is often abbreviated as $f(x) = \ln x$
Converting Between Forms	**Write each equation in logarithmic form.**

1. $e^x = 24$ → $\log_e 24 = x$, $\ln 24 = x$

2. $e^9 = x$

3. $e^{x+5} = 72$ → $\log_e 72 = x+5$, $\ln 72 = x+5$

Write each equation in exponential form.

4. $\ln_e x = 58$ → $e^{58} = x$

5. $\ln 6 = x$

6. $\ln(x-9) = 32$

Simplifying with Properties

Condense each expression into a single logarithm.

7. $\ln 3 + \ln 16$

8. $\ln 63 - 2 \cdot \ln 3$ → $\ln \frac{63}{3^2}$, $\ln \frac{63}{9} = \ln 7$

9. $\frac{1}{3} \cdot \ln 64 + 2 \cdot \ln x$

Expand each logarithm.

10. $\ln 5x$

11. $\ln\left(\frac{a^3}{b}\right)^2 = \ln \frac{a^6}{b^2}$ → $\ln a^6 - \ln b^2$, $6\ln a - 2\ln b$

12. $\ln \sqrt[3]{m^2 n}$ → $\ln (m^2 n)^{\frac{1}{3}}$, $\ln m^{\frac{2}{3}} n^{\frac{1}{3}}$, $\frac{2}{3}\ln m + \frac{1}{3}\ln n$

Solve each equation below. Ch

13. $\ln(4x-27) = \ln(15-2x)$

15. $\ln 72 - \ln 4 = \ln 6 + \ln(a-2)$

17. $\ln_e 8x = 2$ → $e^2 = 8x$, $7.3886 = 8x$, $.9236 = x$

19. $e^x = 57$

21. $\frac{5e^{4n}}{5} = \frac{95}{5}$ → $e^{4n} = 19$, $\log_e 19 = 4n$, $\ln 19 = 4n$, $\frac{2.9444}{4} = \frac{4n}{4}$, $n = .7$

heck for extraneous solutions.

14. $2 \cdot \ln k = \ln(2k + 15)$

$\ln k^2 = \ln(2k+15)$

$k^2 = 2k + 15$

$k^2 - 2k - 15 = 0$

$(k-5)(k+3) = 0$

$k = 5$ ~~$k = -3$~~

16. $2 \cdot \ln(m + 4) = \ln 4$

ve
ing
=
.71828

18. $\ln x - \ln 9 = 7$

20. $e^{y+3} - 6 = 24$

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22. $2e^{c-9} + 3 = 87$