Warmup:

$f(x) = \dfrac{x^3 - 2x^2 - 8x}{3x^2 - 27}$

$\dfrac{x(x^2-2x-8)}{3(x^2-9)}$

$\dfrac{x(x-4)(x+2)}{3(x+3)(x-3)}$

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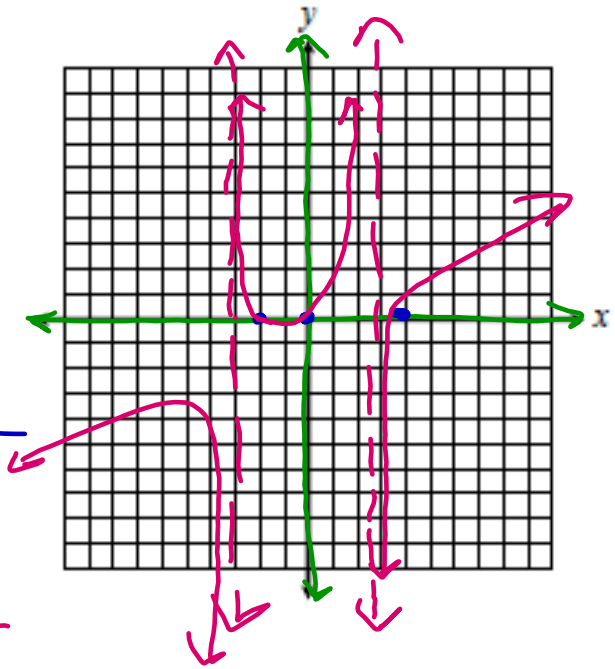

x-int: (0,0), (4,0), (-2,0)

VA: x=-3, x=3

HA: none

Hole: none

Domain: $\{x \mid x \neq -3, 3\}$

Range: $\mathbb{R}$ $(-\infty, \infty)$

Arithmetic Sequence

A sequence where the difference between any two terms is constant.

Common Difference

The numerical difference between two terms. (variable: d)

Examples

Directions: Determine whether the sequence is arithmetic. If yes, identify the common difference and give the next three terms.

1. {5, 8, 11, 14, 17, ...}

+3 +3 +3 +3

20, 23, 26 $d = +3$

2. {23, 16, 9, 2, −5, ...}

3. {1, 2, 3, 5, 8, ...}

4. {−6, −8, −10, −12, −14, ...}

−2 −2 −2 −2 $d = -2$

−16, −18, −20

5. {−20, −10, −5, 0, 5, ...}

+10 +5 +5 +5

No

6. {−35, −23, −11, 1, 13, ...}

Arithmetic Sequence Formula

The n^{th} term of any arithmetic sequence can be found using the formula:

$$a_n = d(n-1) + a_1$$

where a_1 is the first term and d is the common difference

Examples

Directions: Write a rule for each sequence, then find the indicated term.

7. {−5, −1, 3, 7, ...}; a_{24}

+4 +4 +4

$a_n = 4(n-1) + -5$

$a_n = 4n - 4 + -5$

$a_n = 4n - 9$

$a_{24} = 4(24) - 9 = 87$

8. {20, 11, 2, −7, −16 ...}; a_{15}

9. {−6, −9, −12, −15, ...}; a_{36}

−3 −3 −3

$a_n = -3(n-1) + -6$

$a_n = -3n + 3 - 6$

$a_n = -3n - 3$

$a_{36} = -117$

10. {8, 9.5, 11, 12.5, ...}; a_{48}

$a_n = d(n-1) + a_1$

$-7 = d(3-1) + 3$

$-7 = 3d - d + 3$

$-10 = 2d$

$d = -5$

Directions: If the given terms are part of an arithmetic sequence, write a rule for the sequence, then find a_{12}.

11. $a_1 = 3$ and $a_3 = -7$

$a_n = -5(n-1) + 3$

$a_n = -5n + 8$

$a_{12} = -52$

12. $a_1 = -17$ and $a_{24} = 6$

Arithmetic Series

Recall that a series is the sum of a sequence. When finding the sum of an **arithmetic series**, use the following formula:

$$s_n = n\left(\frac{a_1 + a_n}{2}\right)$$

where n is the number of terms, a_1 is the first term, and a_n is the last term.

Examples

Directions: Find the indicated sum for each arithmetic series.

13. $\{2 + 4 + 6 + 8 + ...\}; S_{10}$

$a_{10} = 2(10) + 2$

$a_{10} = 20$

$S_{10} = 10\left(\frac{2 + 20}{2}\right)$

$= 110$

14. $\{(-6) + (-19) + (-32) + (-45) + ...\}; S_8$

15. $\sum_{x=1}^{12}(6x - 4)$

16. $\sum_{m=8}^{17}(-2m + 3)$