

7-6 Solving Rational Equations and Inequalities

Solve Rational Equations A rational equation contains one or more rational expressions. To solve, complete the following steps:

Step 1 State the excluded values (numbers that would make the denominator 0).

Step 2 Multiply each side of the equation by the Least Common Denominator of ALL the denominators.

Step 3 Solve and be sure to exclude any solution that would produce a denominator of 0.

Example: Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$. **Check your solution.**

$$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5} \quad \text{Original equation}$$

$$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right) \quad \text{Multiply each side by } 10(x+1).$$

$$9(x+1) + 2(10) = 4(x+1) \quad \text{Multiply.}$$

$$9x + 9 + 20 = 4x + 4 \quad \text{Distribute.}$$

$$5x = -25 \quad \text{Subtract } 4x \text{ and } 29 \text{ from each side.}$$

$$x = -5 \quad \text{Divide each side by 5.}$$

Check $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$ Original equation

$$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5} \quad x = -5$$

$$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5} \quad \text{Simplify.}$$

$$\frac{2}{5} = \frac{2}{5}$$

Exercises

Solve each equation. Check your solution.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$

2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$

3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$

4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$

5. $\frac{4}{x-1} = \frac{x+1}{12}$

6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$

Solve Rational Inequalities To solve a rational inequality, complete the following steps.

- Step 1** State the excluded values (numbers that would make the denominator 0).
Step 2 Solve the related equation (pretend it isn't an inequality and it has an = sign).
Step 3 Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

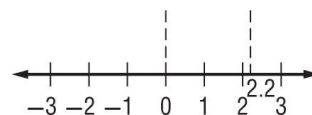
Example: Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$	Related equation
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$	Multiply each side by $15n$.
$10 + 12 = 10n$	Simplify.
$22 = 10n$	Add.
$2.2 = n$	Divide each side by 10.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



Test $n = -1$.

$$-\frac{2}{3} + \left(-\frac{4}{5}\right) \leq \frac{2}{3} \text{ is true.}$$

The solution is $n < 0$ or $n \geq 2.2$.

Test $n = 1$.

$$\frac{2}{3} + \frac{4}{5} \leq \frac{2}{3} \text{ is not true.}$$

Test $n = 3$.

$$\frac{2}{9} + \frac{4}{15} \leq \frac{2}{3} \text{ is true.}$$

Exercises

Solve each inequality. Check your solutions.

1. $\frac{3}{a+1} \geq 3$

2. $\frac{1}{x} \geq 4x$

3. $\frac{1}{2p} + \frac{4}{5p} > \frac{2}{3}$

4. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$

5. $\frac{4}{x-1} + \frac{5}{x} < 2$

6. $\frac{3}{x^2-1} + 1 > \frac{2}{x-1}$