

7-6 Study Guide and Intervention

Transformations of Exponential Functions

Translating an Exponential Function When different operations are performed on the inputs or outputs of an exponential function, the operations performed are called **transformations**.

The **general form** of an exponential function is $f(x) = ab^{x-h} + k$, where a , h , and k are parameters that dilate, reflect, or translate the parent function with base b .

| Translating Exponential Functions | | | |
|--|---|--|---|
| Adding a Constant to $f(x)$, when $f(x) = b^x$ $g(x) = b^x + k$ | | Grouping a Constant with x , when $f(x) = b^x$ $g(x) = b^{(x-h)}$ | |
| $k > 0$; shifts up | $k < 0$; shifts down | $h > 0$; shifts right | $h < 0$; shifts left |
| Example 1: If $f(x) = 2^x$, and $k = 3$, then $g(x) = 2^x + 3$ | Example 2: If $f(x) = 1.4^x$, and $k = -5$, then $g(x) = 1.4^x - 5$ | Example 3: If $f(x) = 0.2^x$, and $h = 1$, then $g(x) = 0.2^{(x-1)}$ | Example 4: If $f(x) = 3^x$, and $h = -2$, then $g(x) = 3^{(x+2)}$ |
| Graph: | Graph: | Graph: | Graph: |
| Description: translates 3 units up | Description: translates 5 units down | Description: translates 1 unit right | Description: translates 2 units left |

Exercises

Write a function $g(x)$ to represent the transformed graph.

- $f(x) = 9^x$ moves 5 units down
- $f(x) = 6^x$ moves 4 units up
- $f(x) = 3^x$ moves 1 unit right
- $f(x) = 2^x$ moves 2 units left
- $f(x) = 0.7^x + 6$ moves 9 units down
- $f(x) = -5^x - 7$ moves 3 units left
- $f(x) = 2.5^x - 3$ moves 4 units right
- $f(x) = 0.1^x + 2$ moves 2 units down

7-6 Study Guide and Intervention *(continued)*

Transformations of Exponential Functions

Dilating and Reflecting an Exponential Function

| Dilating Exponential Functions | | | |
|---|---|---|--|
| Multiplying $f(x)$ by a Positive Constant, when $f(x) = b^x$ $g(x) = ab^x$ | | Multiplying x by a Positive Constant, when $f(x) = b^x$ $g(x) = b^{ax}$ | |
| $a > 1$ | $0 < a < 1$ | $a > 1$ | $0 < a < 1$ |
| Example 1: If $f(x) = 1.5^x$, and $a = 3$, then $g(x) = 3(1.5^x)$ | Example 2: If $f(x) = 2^x$, and $a = 0.3$, then $g(x) = 0.3(2^x)$ | Example 3: If $f(x) = 3^x$, and $a = 2$, then $g(x) = 3^{2x}$ | Example 4: If $f(x) = 1.2^x$, and $a = 0.3$, then $g(x) = 1.2^{0.3x}$ |
| Description: steeper; stretched vertically away from x -axis by a factor of 3 | Description: less steep; compressed vertically towards x -axis by a factor of 0.3 | Description: compressed horizontally towards the y -axis by a factor of 2 | Description: stretched horizontally away from the y -axis by a factor of 1.2 |

| Reflecting Exponential Functions | | | |
|---|--|---|--|
| Multiplying $f(x)$ by -1 | | Multiplying x by -1 | |
| Example 5: If $f(x) = 5^x$; $g(x) = -f(x)$, then $g(x) = -5^x$ | Description: mirror image; reflected across x -axis | Example 6: If $f(x) = 3^x$, and $g(x) = f(-x)$, then $g(x) = 3^{-x}$ | Description: mirror image; reflected across y -axis |

Exercises

Write a function $g(x)$ to represent the transformed graph.

- $f(x) = 5^x$ is stretched vertically away from the x -axis by a factor of 2.
- $f(x) = 4^x$ is compressed vertically toward the x -axis by a factor of $\frac{1}{3}$.
- $f(x) = 1.7^x$ is compressed horizontally towards the y -axis by a factor of 5.

Tell how the transformed function compares to the parent function.

- $f(x) = 3^x$; $g(x) = 3^{3x}$
- $f(x) = 7^x$; $g(x) = 7^{0.25x}$
- $f(x) = 1.25x$; $g(x) = -1.25^x$
- $f(x) = 4^x$; $g(x) = 4^{-x}$
- $f(x) = 0.9^x$; $g(x) = 0.9^{(x+4)} + 2$
- $f(x) = 2.2^x$; $g(x) = -[2.2^x - 7]$