

Warm Up

Find the pattern and the next two numbers in each sequence.

1) 1, 3, 5, 7, 9, 11, ... 2) -2, -4, -6, -8, ...

+2 ; 13, 15

-2 ; -10, -12

3) 0.2, 1, 5, 25, 125, ... 4) 50, 45, 40, 35, ...

*5 ; 625, 3125

-5 ; 30, 25

5) 512, 256, 128, ... 6) 2, 5, 8, 11, 14, ...

÷2 ; 64, 32

3 ; 17, 20

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Key Concepts

Sequence - is an ordered list of numbers.

Term - each number in a sequence.

Explicit Formula - Finding a term without knowing the preceding terms, expresses the n th term in terms of n .

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Example 1

A sequence has an **explicit** formula $a_n = 4n - 3$.
 What are the first five terms of the sequence?
 What is the 19th term?

term
number

$$a_1 = 4(1) - 3 = 1$$

$$a_2 = 4(2) - 3 = 5$$

$$a_3 = 9$$

$$a_4 = 13$$

$$a_5 = 17$$

1, 5, 9, 13, 17, ...

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Example 2

A sequence has an **explicit** formula $a_n = \frac{1}{2}n^2 - 1$.
 What are the first five terms of the sequence?
 What is the 11th term?

$$a_1 = \frac{1}{2}(1) - 1 = -\frac{1}{2}$$

$$a_2 = 1$$

$$a_3 = \frac{7}{2}$$

$$a_4 = 7$$

$$a_5 = \frac{23}{2}$$

$$a_{11} = \frac{119}{2}$$

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Example 3

Given the sequence 1, 2, 4, 8, 16, ... write the explicit formula. Then find the tenth term.

$$a_n = 2^{n-1}$$

$$a_{10} = 512$$

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Example 4

Given the sequence $\frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \dots$ write the explicit formula. Then find the tenth term.

$$a_n = \frac{(-1)^{n-1}}{n+2}$$

$$a_{10} = \frac{-1}{12}$$

$$1^0 = 1$$

$$1^1 = 1$$

$$1^2 = 1$$

$$(-1)^0 = 1$$

$$(-1)^1 = -1$$

$$(-1)^2 = 1$$

$$-1^0 = 1$$

$$-1^1 = -1$$

$$-1^2 = -1$$

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Key Concepts

Recursive Formula - Defines the terms in a sequence by relating each term to the ones before it.

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Example 5

A sequence has a recursive formula

$a_n = 3a_{n-1} + 2$ where $a_1 = \frac{2}{3}$. Write the first five

terms. $a_{1-1} = 0$ $\left(\frac{2}{3}\right)$

$$a_2 = 3\left(\frac{2}{3}\right) + 2 = 4$$

$$a_3 = 3(4) + 2 = 14$$

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Example 6

What is the recursive definition for the sequence 1, 4, 7, 10, 13, ...?

$$a_n = a_{n-1} + 3; a_1 = 1$$

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Example 7

What is the recursive definition for the sequence 2, 3, 5, 9, 19, ...?

$$2a_{n-1} - 1; a_1 = 2$$

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Example 8

Determine whether the following are explicit or recursive. Then find the first three terms.

a) $a_n = \frac{1}{2}a_{n-1} + \frac{2}{3}$ where $a_1 = 1,$

$1, \frac{7}{6}, \frac{5}{4}$

b) $a_n = \frac{n}{(n+1)^2}$

$\frac{1}{4}, \frac{2}{9}, \frac{3}{16}$

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Key Concepts

Arithmetic Sequence - has a difference, d , between two consecutive terms that is constant.

Common Difference - the name of the difference in an Arithmetic Sequence.

2, 5, 8, 11, ... is an arithmetic sequence with

- First term: $a_1 = 2$
- Common difference: $d = 3$
- Number of terms: $n = \text{infinite}$

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Example 1

For the following sequences, identify whether it is an arithmetic sequence, first term (a_1), common difference (d , if applicable), and number of terms (n).

a) $-4, -1, 2, 5, \dots$

$$a_1 = -4$$

$$d = +3$$

$$n = \text{infinite}$$

b) $-10, -6, -2, 0, 2, 6, 10, \dots$

$$a_1 = -10$$

$$d =$$

Not Arithmetic Series

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Key Concepts

Explicit Formula

The n^{th} term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d, \text{ for } n > 1$$

Recursive Formula

A term of an arithmetic sequence with first term $a_1 = a$ and common difference d is given by:

$$a_n = a_{n-1} + d, \text{ for } n > 1$$

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Example 2

Write a rule for the n th term of the sequence 32, 47, 62, 77, ... then find a_{12} .

$$a_n = 15n + 17$$

$$a_2 = 15(2) + 17 \quad a_{12} = 197$$

$$a_3 = 15(3) + 17$$

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Example 3

One of the terms of an arithmetic sequence is $a_8 = 50$. The common difference is $1/4$.

a. Write a rule for the n th term.

$$\frac{1}{4}(n) + 48$$

b. What is the 20 term?

$$\frac{1}{4}(20) + 48 = 53$$

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Example 4

What are the second and third terms of the arithmetic sequence 90, __, __, 12, ...?



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Key Concepts

Arithmetic Mean - the average of any two numbers.

$$\frac{x+y}{2}$$

- This is used to find a missing term if you know the term before and the term after.

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