Solving Exponential Equations

Exponential equations have a variable in the exponent

We can solve these equations by rewriting both sides of the equation with a common base.

Common Base Practice

Rewrite the following with base 2:

$32 = 2^5$

$4^x = (2^2)^x = 2^{2x}$

$16^{2x} = (2^4)^{2x} = 2^{8x}$

$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$\frac{1}{4^{2x}} = 4^{-2x} = (2^2)^{-2x} = 2^{-4x}$

Steps for Solving Equations

1. Rewrite both sides of the equation with a common base.
2. Set the exponents on each side equal to one another.
3. Solve the equation.
4. Check your answer!

$$8^x = 4$$
$$(2^3)^x = 2^2$$
$$2^{3x} = 2^2$$
$$3x = 2$$
$$x = 2/3$$

27
9 · 3
3 3
3^3

$$27^x = \frac{1}{81}$$
$$(3^3)^x = \frac{1}{3^4}$$
$$3^{3x} = 3^{-4}$$
$$3x = -4$$
$$x = -4/3$$

81
9 9
3 3 3 3
3^4

$$5^{3x} = \frac{1}{125}$$
$$5^{3x} = \frac{1}{5^3}$$
$$5^{3x} = 5^{-3}$$
$$3x = -3$$
$$x = -1$$

$$25^{y-3} \le \left(\frac{1}{125}\right)^{y+2}$$
$$(5^2)^{y-3} \le \left(\frac{1}{5^3}\right)^{y+2}$$
$$5^{2y-6} \le (5^{-3})^{y+2}$$
$$5^{2y-6} \le 5^{-3y-6}$$
$$2y-6 \le -3y-6$$
$$+3y \qquad +3y$$
$$5y-6 \le -6$$
$$+6 \qquad +6$$
$$5y \le 0 \qquad y \le 0$$

$$3^{1-2x} = 243$$
$$3^{1-2x} = 3^5$$
$$1-2x = 5$$
$$-2x = 4$$
$$x = -2$$

$$9^{2x-5} = 27$$
$$(3^2)^{2x-5} = 3^3$$
$$3^{2(2x-5)} = 3^3$$
$$3^{4x-10} = 3^3$$
$$4x-10 = 3$$
$$4x = 13$$
$$x = 13/4$$

$$5^{3x-8} = 25^{2x}$$
$$5^{3x-8} = (5^2)^{2x}$$
$$5^{3x-8} = 5^{4x}$$
$$3x-8 = 4x$$
$$-8 = x$$
$$x = -8$$

Mr. Foster is starting a new job. His salary for the first year is \$30,000. He will receive a 5% raise each year after that. Write a formula to define Mr. Foster's salary, s, for the nth year.

$$S = 30000(1+.05)^{n-1}$$

How much will Mr. Foster make after 7 years?

$$S = 30000\,(1+.05)^{6}$$
$$= 40,202.87$$

Janet purchased a new car for \$25,000. The moment, she drove the car off the lot, it began depreciating 15% per year. Write a formula to define the value of Janet's car, v, after n years.

$$V = 25,000(1-.15)^{n}$$

How much is Janet's car worth after 4 years?

$$= 25,000(.85)^{4}$$
$$= 13,050$$

You take a 325 milligram dosage of ibuprofen. During each subsequent hour, the amount of medication in your blood-stream decreases by about 29% each hour. Write an exponential equation giving the amount y (in milligrams) of ibuprofen in your bloodstream t hours after the initial dose.

$$y = 325(1-.29)^{t}$$

How much ibuprofen will remain in your bloodstream after 3 hours?

$$= 325(.71)^{3}$$
$$= 116\text{mg}$$

EXPONENTIAL FUNCTIONS

KEY

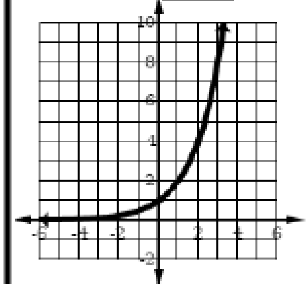
$$y = a \cdot b^x$$

An exponential function is defined by $f(x) = a \cdot b^x$ with base b.

- The base, b, is a constant such that $b > 0$ and $b \neq 1$.
- $a \neq 0$
- The exponent, x, is a variable. x is any real number.

WOW!

Example 1: $y = 2^x$



Fill in the table:

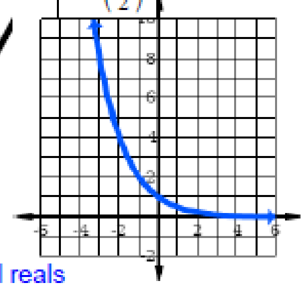
x	$y = f(x)$
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

The domain is __All reals__

The range is __$(0,\infty)$__

Example 2: Finish the table, graph the function, and answer the questions below.

$y = \left(\frac{1}{2}\right)^x$



x	$y = f(x)$
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4

The domain is __All reals__

The range is __$(0,\infty)$__

PROPERTIES OF EXPONENTIAL GRAPHS $y = a \cdot b^x$

The domain is __$(-\infty,\infty)$__

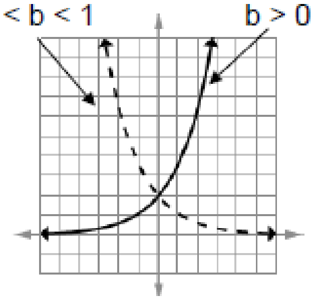
The range is __$(0,\infty)$__

The graphs have a y-intercept at __(0,a)__

When $b > 0$, the graph __Increases__

When $0 < b < 1$ the graph __Decreases__

The graphs have an asymptote at __y = 0 x-axis__



BAM!

Exponential Functions

Exponential Growth	Exponential Decay

KEY

Exponential Growth

$y = a \cdot b^x$

a is the initial amount.

b is the growth factor when b >1

$y = a(1+r)^x$

If we rewrite b as (1 + r), we can determine the **GROWTH RATE**, r. r is usually a percentage and written as a decimal

Example 3:

$y = 2(1.43)^x$

Growth factor__1.43___

Growth rate___.43____

Initial value ___2_____

Exponential Decay

$y = a \cdot b^x$

a is the initial amount.

b is the decay factor when 0 < b < 1

$y = a(1-r)^x$

If we rewrite b as (1 - r), we can determine the **DECAY RATE**, r. r is usually a percentage and written as a decimal

Example 4: $y = 300(.75)^x$

Decay factor_.75_____

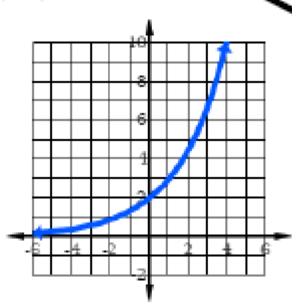
Decay rate___.25____

Initial value __300_____

Example 6: Given the following growth function, use the table to help sketch the graph and answer the questions below.

$y = 2(1.5)^x$

x	$y = f(x)$
-2	8/9
-1	4/3
0	2
1	3
2	4.5
3	6.75



Decay factor___1.5______

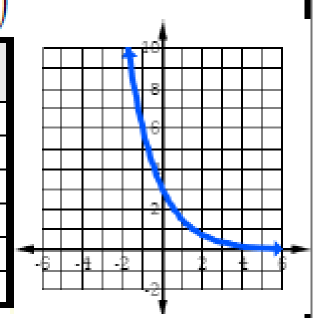
Decay rate_____.5_____

Initial value ___2______

Example 6: Given the following decay function, use the table to help sketch the graph and answer the questions below.

$y = 2\left(\frac{1}{3}\right)^x$

x	$y = f(x)$
-2	18
-1	6
0	2
1	2/3
2	2/9
3	2/27



Decay factor___1/3_____

Decay rate____ 2/3 ____

Initial value ____2______