

## 5-Minute Check

Over Lesson 5-2

Given  $f(x) = 3x + 2$  and  $g(x) = 2x^2 - 1$ , find  $[f \circ g](x)$  and  $[g \circ f](x)$ .

$$g(f(x))$$

$$g[f(x)] = 18x^2 + 24x + 7$$

$$f(g(x))$$

$$f[g(x)] = 6x^2 - 1$$

Let  $f(x) = x - 3$  and  $g(x) = x^2$ . Find  $(f \circ g)(1)$ ?

$$f(x^2 - 3)$$

$$1 - 3$$

$$-2$$

$$f(1)$$



## **Mathematical Practices**

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

## **Content Standards**

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.BF.4.a Find inverse functions. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.

 **Key Concept** Inverse Relations

**Words** Two relations are inverse relations if and only if whenever one relation contains the element  $(a, b)$ , the other relation contains the element  $(b, a)$ .

**Example**  $A$  and  $B$  are inverse relations.

$$A = \{(1, 5), (2, 6), (3, 7)\} \quad B = \{(5, 1), (6, 2), (7, 3)\}$$

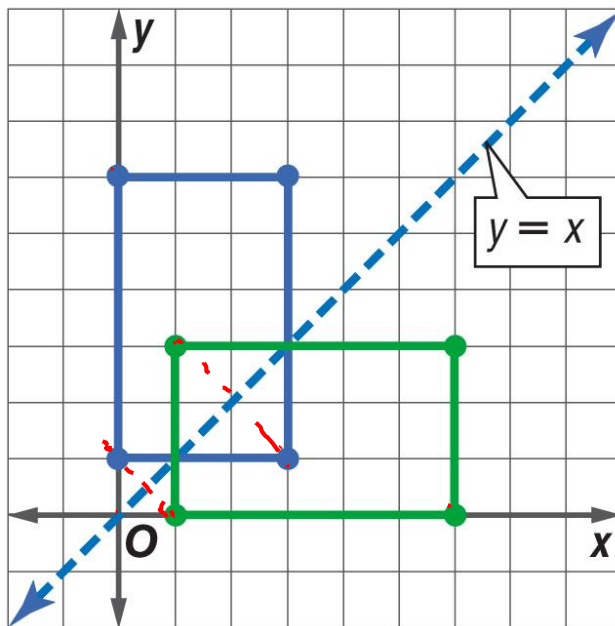
**GEOMETRY** The ordered pairs of the relation  $\{(1, 3), (6, 3), (6, 0), (1, 0)\}$  are the coordinates of the vertices of a rectangle. Find the inverse of this relation.


$$\{(3, 1), (3, 6), (0, 6), (0, 1)\}$$

## Example 1

### Find an Inverse Relation

Plotting the points shows that the ordered pairs also describe the vertices of a rectangle. Notice that the graph of the relation and the inverse are reflections over the graph of  $y = x$ .



 **Key Concept** Property of Inverses

Words

If  $f$  and  $f^{-1}$  are inverses, then  $f(a) = b$  if and only if  $f^{-1}(b) = a$ .

Example

Let  $f(x) = x - 4$  and represent its inverse as  $f^{-1}(x) = x + 4$ .

Evaluate  $f(6)$ .

$$f(x) = x - 4$$

$$f(6) = 6 - 4 \text{ or } 2$$

Evaluate  $f^{-1}(2)$ .

$$f^{-1}(x) = x + 4$$

$$f^{-1}(2) = 2 + 4 \text{ or } 6$$

Because  $f(x)$  and  $f^{-1}(x)$  are inverses,  $f(6) = 2$  and  $f^{-1}(2) = 6$ .

$f^{-1}$  is inverse of  $f$

An Inverse Function is found by exchanging domain (x) and range (y).

$$f(x) = x - 4$$

$$y = x - 4$$

$$x = y - 4$$

$$-y = -x - 4$$

$$y = x + 4$$

**Example 2**

**Find and Graph an Inverse**

Find the inverse of  $f(x) = -\frac{1}{2}x + 1$ .

Then graph the function and its inverse.

$$y = -\frac{1}{2}x + 1$$

$$x = -\frac{1}{2}y + 1$$

$$\Rightarrow (x - 1 = -\frac{1}{2}y)^{-2}$$

$$-2x + 2 = y$$

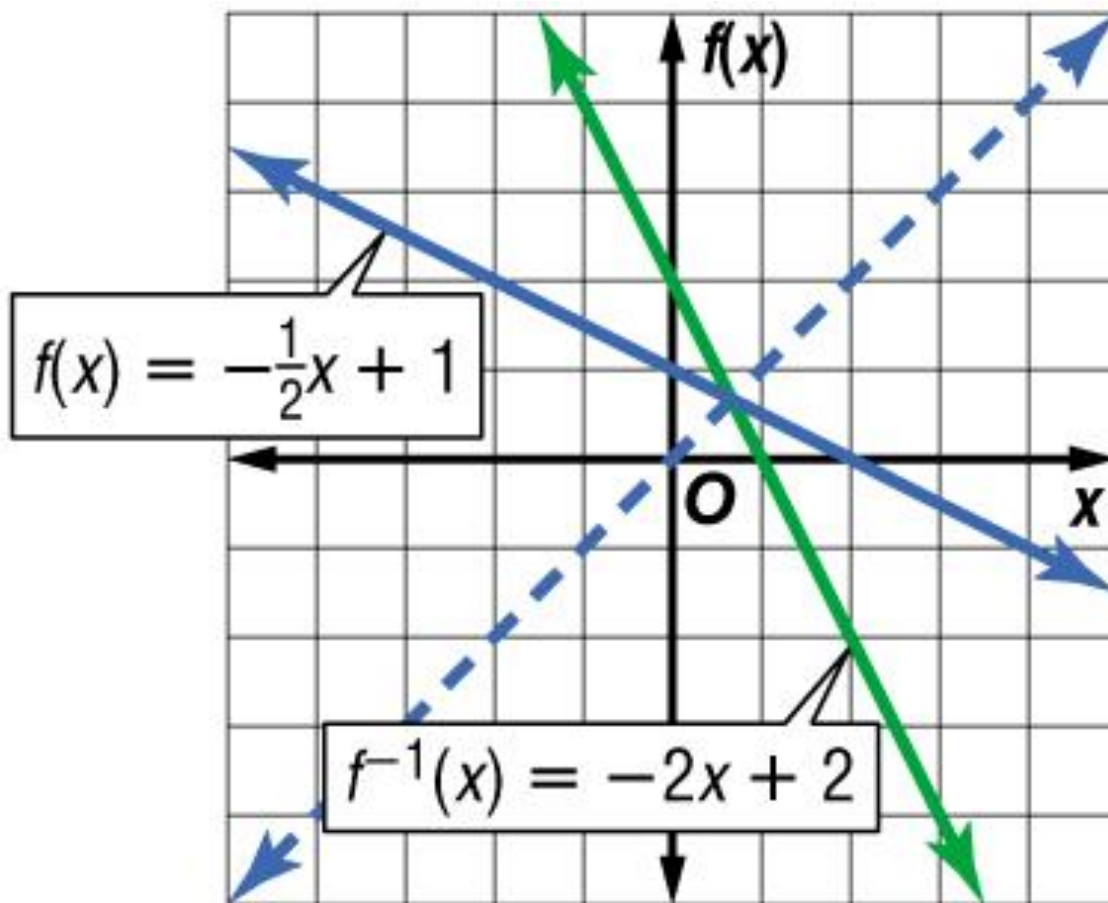
$$f^{-1}(x) = -2x + 2$$

**Step 1** Replace  $f(x)$  with  $y$  in the original equation.

**Step 2** Interchange  $x$  and  $y$ .

**Step 3** Solve for  $y$ .

**Step 4** Replace  $y$  with  $f^{-1}(x)$ .

**Example 2****Find and Graph an Inverse**



To determine whether a pair of functions are inverses, you can use composition of functions:

## Key Concept Inverse Functions

**Words** Two functions  $f$  and  $g$  are inverse functions if and only if both of their compositions are the identity function.

**Symbols**  $f(x)$  and  $g(x)$  are inverses if and only if  $[f \circ g](x) = x$  and  $[g \circ f](x) = x$ .

$$f(x) = 3x + 9 \quad \text{and} \quad g(x) = \frac{1}{3}x - 3$$

$f \circ g(x)$      $f(g(x))$

$$3\left(\frac{1}{3}x - 3\right) + 9$$

$$x - 9 + 9$$

$$x$$

$g \circ f(x)$      $g(f(x))$

$$\frac{1}{3}(3x + 9) - 3$$

$$x + 3 - 3$$

$$x$$