Lesson Menu

5-Minute Check

Over Lesson 7–2

1 Simplify $\frac{7^2}{7^4}$.

A. 49

B. $\frac{1}{7}$

C. $\frac{1}{49}$

D. $\frac{1}{343}$

5-Minute Check

Over Lesson 7–2

2 Simplify $\frac{-9n^6}{18n^2}$. Assume that the denominator does not equal zero.

A. $\frac{-n^4}{18}$

B. $2n^4$

C. $\frac{n^4}{2}$

D. $\frac{-n^4}{2}$

5-Minute Check Over Lesson 7–2

3 Simplify $\left(\frac{2}{5}\right)^{-2}$.

A. $\frac{25}{4}$

B. $-\frac{4}{25}$

C. $\frac{4}{5}$

D. $\frac{4}{25}$

5-Minute Check

Over Lesson 7–2

4 Simplify $\frac{(3x^2)^2 y^3}{24x^{-2}y}$. Assume that the denominator does not equal zero.

A. $\frac{x^6y^2}{8}$

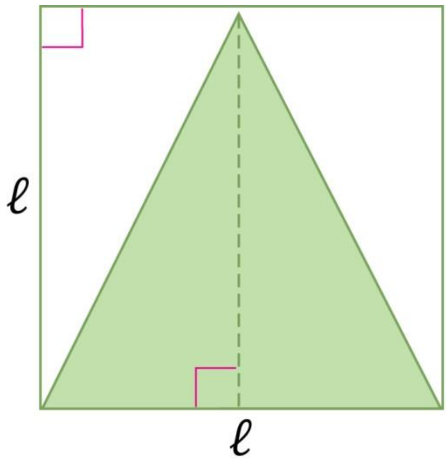
B. $\frac{3x^6y^2}{8}$

C. $\frac{3x^4y^3}{8y}$

D. $\frac{3x^4y^5}{8}$

5-Minute Check Over Lesson 7–2

5 What is the ratio of the area of the square to the area of the triangle?



A. 2 to 1

B. 1 to 1

C. 4 to 1

D. 3 to 1

5-Minute Check

Over Lesson 7–2

6 Simplify $\left(\frac{2^3xy^3}{8x^2y}\right)^2$. Assume that the denominator does not equal zero.

A. $\frac{y^3}{2x^2}$

B. $\frac{y^2}{x}$

C. $\frac{y^4}{x}$

D. $\frac{y^4}{x^2}$

MP

Mathematical Practices

5 Use appropriate tools strategically.

Content Standards

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Then

You used the laws of exponents to find products and quotients of monomials.

Now

- Evaluate and rewrite expressions involving rational exponents.
- Solve equations involving expressions with rational exponents.

New Vocabulary

- rational exponent
- cube root
- *n*th root
- exponential equation

Key Concept $b^{\frac{1}{n}}$

Words	For any nonnegative real number b, $b^{\frac{1}{2}} = \sqrt{b}$.
Examples	$16^{\frac{1}{2}} = \sqrt{16}$ or 4 $\quad 38^{\frac{1}{2}} = \sqrt{38}$

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

A. $81^{\frac{1}{2}}$

$81^{\frac{1}{2}} = \sqrt{81}$ Definition of $b^{\frac{1}{2}}$

$= 9$ Simplify.

Answer: 9

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

B. $\sqrt{38}$

$\sqrt{38} = 38^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

Answer: $38^{\frac{1}{2}}$

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

C. $12m^{\frac{1}{2}}$

$12m^{\frac{1}{2}} = 12\sqrt{m}$ Definition of $b^{\frac{1}{2}}$

Answer: $12\sqrt{m}$

Example 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

D. $\sqrt{32w}$

$\sqrt{32w} = (32w)^{\frac{1}{2}}$ Definition of $b^{\frac{1}{2}}$

Answer: $(32w)^{\frac{1}{2}}$

Example 1 Guided Practice

Write $16^{\frac{1}{2}}$ in radical form.

A. 16

B. 8

C. 4

D. 2

KeyConcept nth Root

Words	For any real numbers a and b and any positive integer n, if $a^n = b$, then a is an nth root of b.
Symbols	If $a^n = b$, then $\sqrt[n]{b} = a$.
Example	Because $2^4 = 16$, 2 is a fourth root of 16; $\sqrt[4]{16} = 2$.

Example 2 *n*th Roots

A. Simplify $\sqrt[4]{256}$.

$$\sqrt[4]{256} = \sqrt[4]{4 \bullet 4 \bullet 4 \bullet 4}$$

$$= 4$$

Answer: 4

Example 2 *n*th Roots

B. Simplify $\sqrt[6]{15,625}$.

$$\sqrt[6]{15,625} = \sqrt[6]{5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5}$$

$$= 5$$

Answer: 5

Example 2 Guided Practice

Simplify $\sqrt[4]{2401}$.

A. 343

B. 81

C. 49

D. 7

KeyConcept $b^{\frac{1}{n}}$

Words For any positive real number b and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$.

Example $8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$ or 2

Example 3

A. Simplify $1331^{\frac{1}{3}}$.

$1331^{\frac{1}{3}} = \sqrt[3]{1331}$ $b^{\frac{1}{n}} = \sqrt[n]{b}$

$= \sqrt[3]{11 \bullet 11 \bullet 11}$ $11^3 = 1331$

$= 11$ Simplify.

Answer: 11

Example 3

B. Simplify $2401^{\frac{1}{4}}$.

$$2401^{\frac{1}{4}} = \sqrt[4]{2401} \qquad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$= \sqrt[4]{7 \bullet 7 \bullet 7 \bullet 7} \qquad 7^4 = 2401$$

$$= 7 \qquad \text{Simplify.}$$

Answer: 7

Example 3 Guided Practice

Simplify $4096^{\frac{1}{4}}$.

A. 4

B. 8

C. 16

D. 32

KeyConcept $b^{\frac{m}{n}}$

Words For any positive real number b and any integers m and $n > 1$,

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m \text{ or } \sqrt[n]{b^m}.$$

Example $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2$ or 4

Example 4

Evaluate $b^{\frac{m}{n}}$ Expressions

A. Simplify $32^{\frac{2}{5}}$.

$32^{\frac{2}{5}} = \left(\sqrt[5]{32}\right)^2$ — Rewrite the problem using the exponent property $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= 2^2$ — The 5th root of 32 is 2.

$= 4$ — Simplify.

Answer: 4

B. Simplify $81^{\frac{5}{2}}$.

$81^{\frac{5}{2}} = \left(\sqrt{81}\right)^5$ Rewrite the problem using the exponent property $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$

$= 9^5$ The square root of 81 is 9.

$= 59{,}049$ Simplify.

Answer: 59,049

Example 4

Guided Practice

Simplify $36^{\frac{3}{2}}$.

A. 216

B. 36

C. 18

D. 6

KeyConcept Power Property of Equality

Words For any real number $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

Examples If $5^x = 5^3$, then $x = 3$. If $n = \frac{1}{2}$, then $4^n = 4^{\frac{1}{2}}$.

Example 5 Solve Exponential Equations

A. Solve $9^x = 729$.

$9^x = 729$	Original equation
$9^x = 9^3$	Rewrite 729 as 9^3.
$x = 3$	Power Property of equality

Answer: 3

Example 5 Solve Exponential Equations

B. Solve $16^{2x-1} = 8$.

$16^{2x-1} = 8$	Original equation
$(2^4)^{2x-1} = 2^3$	Rewrite 16 as 2^4 and 8 as 2^3.
$2^{8x-4} = 2^3$	Power of a Power, Distributive Property
$8x - 4 = 3$	Power Property of Equality
$8x = 7$	Add 4 to each side.
$x = \frac{7}{8}$	Divide each side by 8.

Answer: $\frac{7}{8}$

Example 5

Guided Practice

Solve $9^{3x + 1} = 27^4$.

A. $\frac{1}{9}$

B. $\frac{5}{3}$

C. $\frac{11}{6}$

D. 2

$3^2 = 9$ $\quad$ $3^3 = 27$

$(3^2)^{3x+1} = (3^3)^4$

$3^{2(3x+1)} = 3^{(3)(4)}$

$3^{6x+2} = 3^{12}$

$6x + 2 = 12$

$6x = 10$

$\frac{10}{6} =$

Real-World Example 6 — Solve Exponential Equations

BIOLOGY **The population *p* of a culture that begins with 40 bacteria and doubles every 8 hours can be modeled by** $p = 40(2)^{\frac{t}{8}}$**, where *t* is time in hours. Find *t* if** $p = 20{,}480$**.**

$p = 40(2)^{\frac{t}{8}}$	Original equation
$20{,}480 = 40(2)^{\frac{t}{8}}$	$p = 20{,}480$
$512 = 2^{\frac{t}{8}}$	Divide each side by 40.

Real-World Example 6 — Solve Exponential Equations

$2^9 = 2^{\frac{t}{8}}$	Rewrite 512 as 2^9.
$9 = \frac{t}{8}$	Power Property of Equality
$72 = t$	Multiply each side by 8.

Answer: **72 hours**

Real-World Example 6 — Guided Practice

NEWS **The number of hits to a news story online can be modeled by** $h = 6(2)^{\frac{4t}{3}}$**, where** t **is time in hours. Find** t **if** $h = 24{,}576$**.**

A. 6

B. 8

C. 9

D. 12